Ratio

Component Knowledge

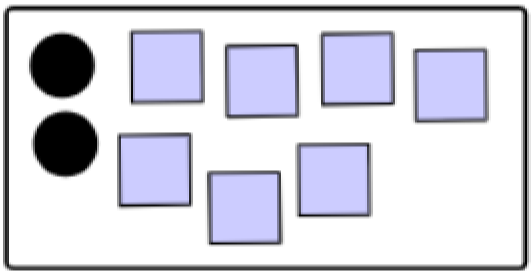
- Write a ratio
- Simplify a ratio
- Sharing into a ratio given the total
- Sharing into a ratio given a part of the ratio.
- Sharing into a ratio given the difference between two parts

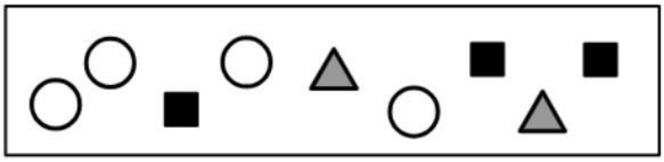
Key Vocabulary

Ratio	The relative sizes of two or more values.
Simplify	Reducing the ratio into a simpler form by dividing by a common factor .
Share	To split into equal parts or groups.
Equivalent	Equal in amount or value but looks different.
Part	This is the numeric value An equal amount that, when combined with others creates the whole.

Write a ratio

When writing a ratio, the order is important. Each number must be separated by a colon ":"



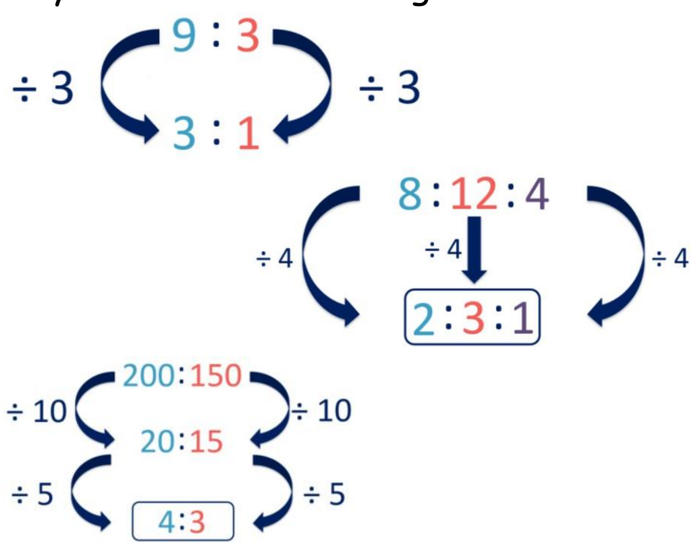


Ratio of circles to squares is 2 : 7
This means that for every 2 circles there are 7 squares

Ratio of circles to triangles to squares is 4 : 2 : 3
This means that for every 4 circles there are 2 triangles and 3 squares

Simplify ratios

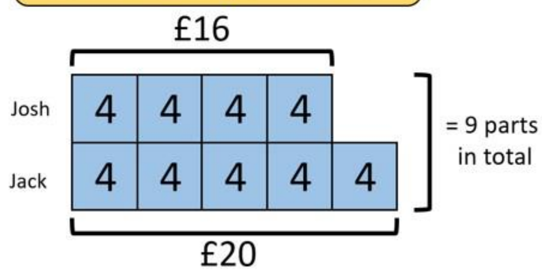
To simplify a ratio, divide all numbers in the ratio by the same amount. You may need to do it in stages.



Sharing into a ratio given a total

Josh and Jack have **£36.**
They divided it in the ratio **4 : 5**
How much did they each get?

Draw a **Bar Model** to calculate how much **one part** is worth.

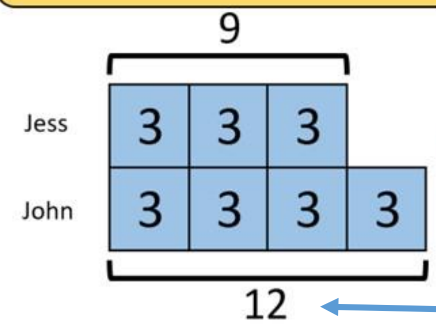


£36 ÷ 9 = £4 per part
Josh - £4 × 4 = £16
Jack - £4 × 5 = £20

Sharing into a ratio given a part

Jess and John shared **some marbles** in the ratio **3 : 4** John got **12 marbles.**
How many marbles were there in total?

Draw a **Bar Model** to calculate how much **one part** is worth.



= 21 marbles in total

4 parts = 12 marbles

12 ÷ 4 = 3 marbles per part

3 marbles per part × 7 parts = 21 marbles in total

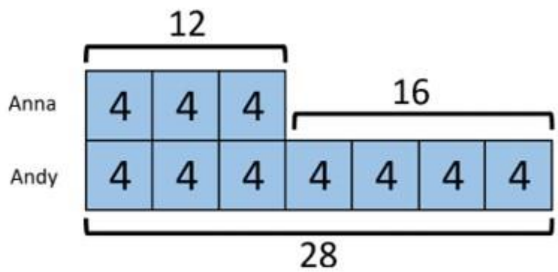
Sharing into a ratio given the difference between two parts

Anna and Andy shared **some sweets** in the ratio **3 : 7.** Andy got **16 more** than Anna.
How many sweets were there in total?

Draw a **Bar Model** to calculate how much **one part** is worth.

16 ÷ 4 =
4 per part

e



= 40 sweets in total

16 ÷ 4 = 4 sweets per part

4 sweets per part × 10 parts = 40 sweets in total

Online clips

M885, M801, M525, M543

Proportion

Component Knowledge

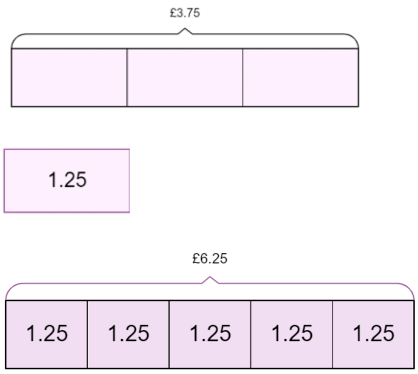
- Find the value of 1 item (unitary method)
- Use proportion to work out which item is best value for money
- Use proportion to solve problems involving exchange rates
- Use proportion to solve problems involving recipes

Key Vocabulary

Proportion	2 or more quantities that change by a related amount in the same ratio.
Exchange rate	The amount of money in a different currency that your currency will buy or sell for.
Best buy	Comparing the cost of 2 or more items and interpreting the values.
Unitary method	Finding the value of 1 item.
Direct proportion	A relationship between two quantities such that as one increases, the other increase (or as one decrease, the other decreases) at the same rate.

Unitary method

Finding the value of a single unit and then finding the necessary value by multiplying the single unit value.



Example

If 3 ice creams cost £3.75, how much does 1 ice cream cost?

£3.75 ÷ 3 = £1.25. 1 ice cream costs £1.25.

How much do 5 ice creams cost? (use the cost of 1 ice cream to find this)

£1.25 × 5 = £6.25. 5 ice creams cost £6.25.

Best buys

Find the unit cost by dividing the price by the quantity (unitary method). The lowest number is the best value.

Shop A	Shop B
4 cans for £1.20	3 cans for 93p
£1.20 ÷ 4	£0.93 ÷ 3
1 can is £0.30 Or 30p	1 can is £0.31 Or 31p

Cost per item

Best value is the most product for the lowest price per unit

Shop A is the better value.

You can also compare using multiples. Multiply both amounts until you have the same number of items (12 in this case). Then compare the costs to find the lowest.

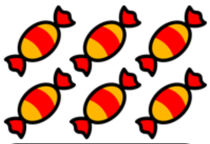
Eat Fresh

4 for 46p

46p × 3 = £1.38 for 12

CHEAPEST

Max-Mart



6 for 75p

75p × 2 = £1.50 for 12

× 1.5

£1 = $1.50

÷ 1.5

For every £1, you can buy $1.50 US dollars
This is the price of one pound, expressed in dollars
i.e. the £/$ exchange rate

To change an amount of £ into $, multiply by 1.50

To change an amount of $ into £, divide by 1.50

Exchange rates

Examples

Change £200 into US dollars. £200 × $1.5 = $300

Change $75 into British Pounds $75 ÷ $1.5 = £50

A watch costs £45 in Manchester. The same watch costs $68 in New York. In which place is the watch cheaper?
(Both prices need to be in the same currency)

£45 × $1.5 = $67.50. (Both in US dollars)

The watch is cheaper in Manchester.

Recipes

Non calculator

A recipe to make **10 cupcakes:**

100 g of butter
100 g of sugar
100 g of flour
2 eggs

How much of each ingredient is needed to make **15 cupcakes?**

To get from 10 to 15, divide by 2 and then multiply by 3

÷ 2

5 cupcakes:

50 g of butter
50 g of sugar
50 g of flour
1 egg

× 3

15 cupcakes:

150 g of butter
150 g of sugar
150 g of flour
3 eggs

Calculator

A recipe to make **10 cupcakes:**

100 g of butter
100 g of sugar
100 g of flour
2 eggs

How much of each ingredient is needed to make **15 cupcakes?**

Use the unitary method, divide by 10 to find how much 1 cupcake needs and then multiply by 15

1 cupcake:

10g of butter
10g of sugar
10g of flour
0.2 of an egg

15 cupcakes:

150 g of butter
150 g of sugar
150 g of flour
3 eggs

Online clips

M478, M681, U610

Area of 2-D shapes

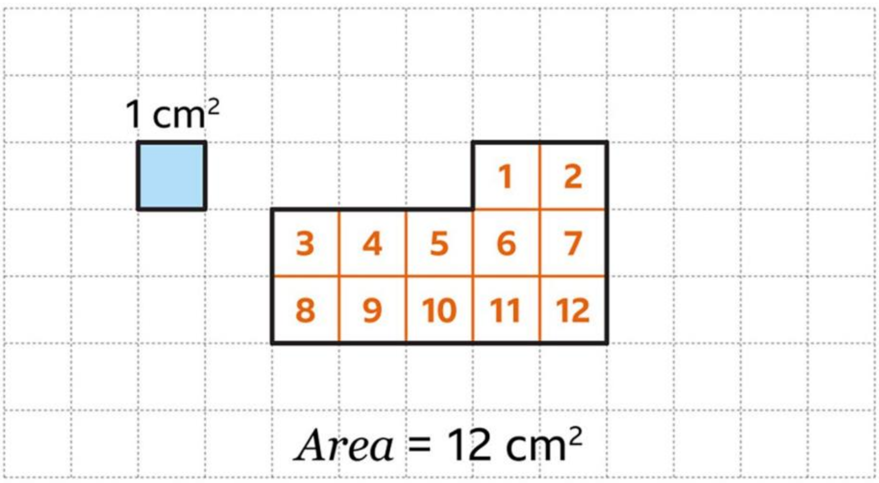
Component Knowledge

- Identify the relevant dimensions
- Identify the correct formula for area
- Use the correct formula to calculate the area of rectangles, triangles, parallelograms and trapeziums.
- Express the answer in the correct units

Key Vocabulary

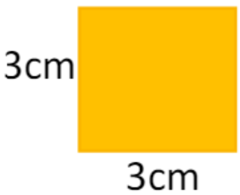
Area	The amount of squared units that fit inside a shape
Dimension	The lengths of the sides of the shape
Unit of measure	This can be length (cm, mm, m) or area (cm^2, mm^2)
Compound shape	A 2-D shape composed of key 2-D shapes

Area is how much space fits inside a shape. We usually measure it in cm^2, this means how many 1cm squares can fit inside the shape.

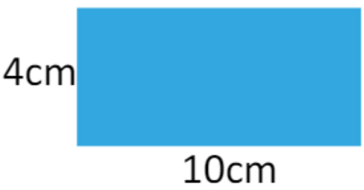


Squares and rectangles:

The formula is the same for both shapes: **A = Length x Width**



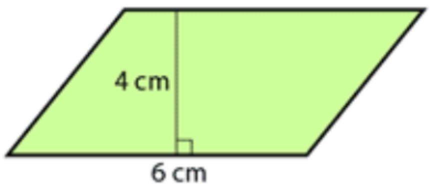
$A = 3 \times 3$
$= 9cm^2$



$A = 10 \times 4$
$= 40cm^2$

Parallelograms:

The formula is similar to a rectangle but instead of width we use the height. **A = Length x Height**



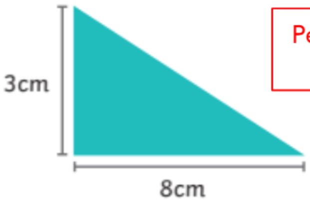
$A = 6 \times 4$
$= 24cm^2$

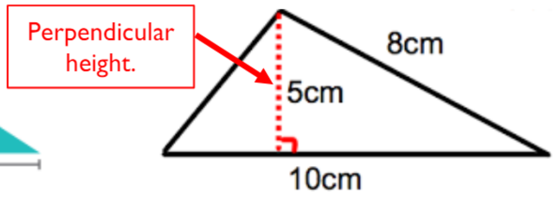
Sometimes the length is referred to as the base.

Triangles: To find the area of a triangle we use the following formula:

$$\textbf{Area} = \frac{\text{Base x perpendicular height}}{2}$$

The formula is very similar to a rectangle but we must divide by 2 because a triangle is half the size of a rectangle.





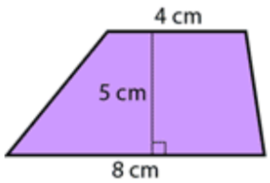
$$\text{Area} = \frac{8 \times 3}{2} = 12\text{cm}^2$$

$$\text{Area} = \frac{10 \times 5}{2} = 25\text{cm}^2$$

Trapeziums: To find the area of a trapezium we use the following formula:

$$\textbf{Area} = \frac{(a+b)}{2} \times h$$

Where a and b are the parallel sides and h is the height.



Area= 4 + 8 = 12 ← Add the parallel sides.

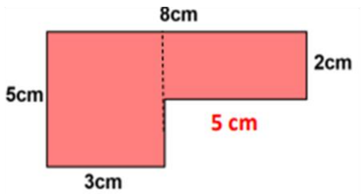
12 ÷ 2 = 6 ← Divide the total by 2.

6 x 5 = 30cm² ← Multiply by the height.

Compound shape example

A compound shape is a shape made up of other shapes.

Example 1

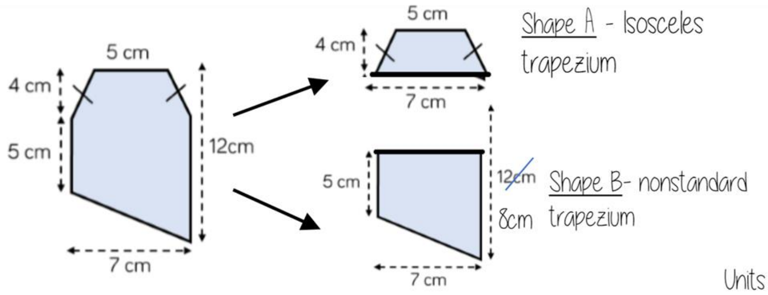


$$Area = (5 \times 3) + (2 \times 5)$$
$$= 25cm^2$$

You must determine any missing dimensions,

e.g. 8 – 3 = 5cm

Example 2



Shape A + Shape B = total area

$\frac{(5+7) \times 4}{2} + \frac{(5+8) \times 7}{2} = 24 + 45.5 = 69.5cm^2$ ← Units

Online clips

M900, M390, M291, M610, M269, M996

Terms and notations of 3D shapes

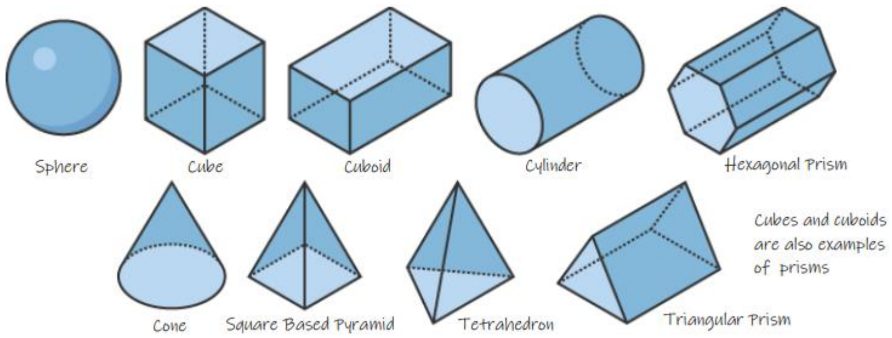
Component Knowledge

- Be able to name 3D shapes
- Identify edges, faces and vertices on 3D shapes
- Recognise nets of 3D shapes

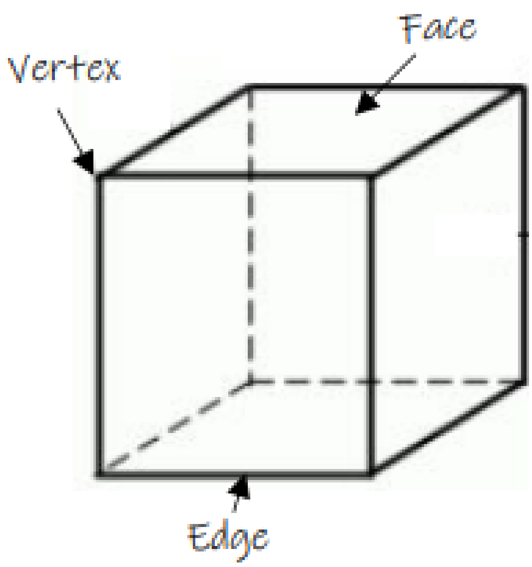
Key Vocabulary

3 dimensional	Having 3 dimensions such as height, width and depth
Vertices	Where edges meet to form a point
Edge	Where two faces meet
Face	A flat surface
Net	A flat 2D shape which can be folder to create a 3D shape
Prism	A type of 3D shape with two ends that are the same shape and size.

Names of 3D shapes



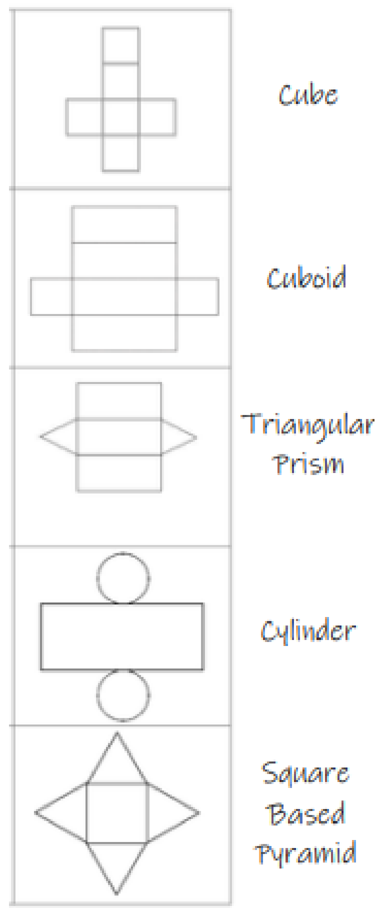
Properties of 3D shapes



A cube has 6 faces, 8 vertices and 12 edges.

You may not be able to see all the faces, edges and vertices on a shape but the hidden ones are still counted

Nets of 3D shapes



A net only works if you have no overlapping pieces once your 3D shape has been formed

Online clips

Q675, Q711, Q971

Volume

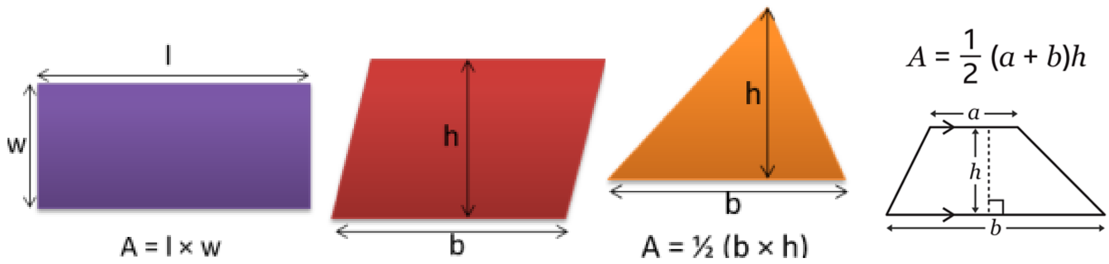
Component Knowledge

- To be able to calculate the volume of a prism
- To be able to calculate the volume of a sphere
- To be able to calculate the volume of a cone
- To be able to calculate the volume of a pyramid

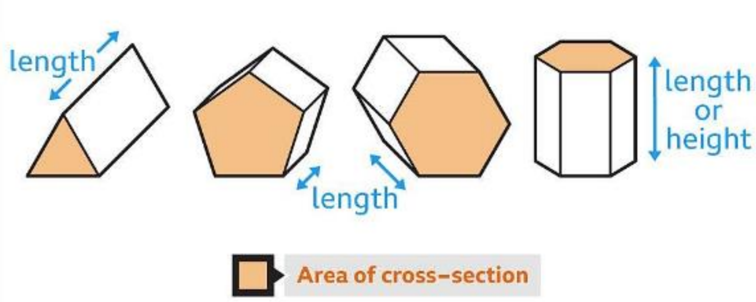
Key Vocabulary

Volume	The amount of space that a shape occupies
Prism	A prism is a solid object with identical ends and flat faces. And the same cross section all along its length.
Length	How long a shape is.
Cross-section	A cross section is the shape made by cutting straight across an object
Face	The flat part of a 3D solid.
Pyramid	A 3D shape with a flat base and its sides meet at a single vertex. It's volume is a third of the volume of its prism.

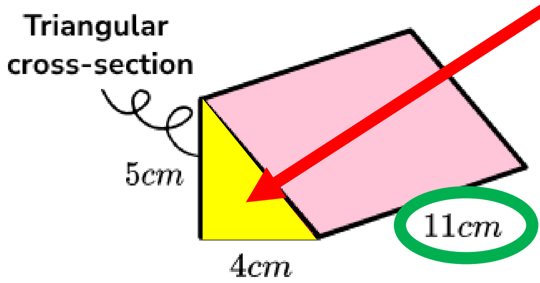
Area – recap



Volume of a prism = Area of the cross section × Length



Example of volume of a prism



First start by finding the area of the cross section, which in this example is a triangle

$$Area\ of\ triangle = \frac{1}{2}(b \times h)$$

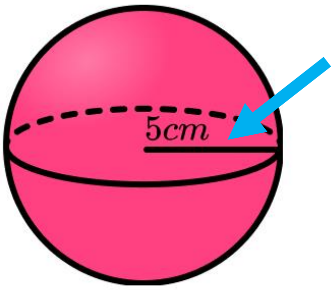
$$Area = \frac{1}{2}(4 \times 5) = \frac{1}{2}(20) = 10cm^2$$

$$volume = area\ of\ cross \times length$$

$$volume = 10 \times 11 = 110cm^3$$

Volume of a sphere

$Volume = \frac{4}{3}\pi r^3$



Here the radius is 5cm

$Volume = \frac{4}{3} \times \pi \times r^3$

$Volume = \frac{4}{3} \times \pi \times 5^3$

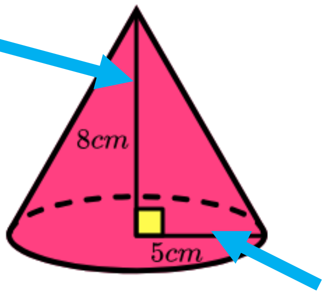
$Volume = \frac{4}{3} \times \pi \times 125$

$Volume = 523.6\ cm^3$

Volume of a cone

$Volume = \frac{1}{3}\pi r^2 h$

Here the height is 8cm



Here the radius is 5cm

$Volume = \frac{1}{3} \times \pi \times r^2 \times h$

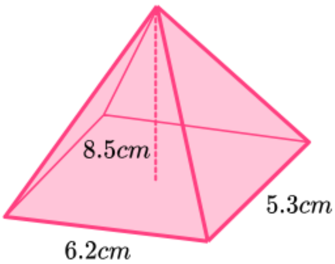
$Volume = \frac{1}{3} \times \pi \times 5^2 \times 8$

$Volume = \frac{1}{3} \times \pi \times 25 \times 8$

$Volume = 209.4\ cm^3$

Volume of a pyramid

$$Volume = \frac{1}{3} \times base\ area \times height$$

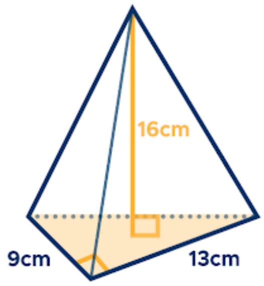


Here the base is a rectangle

$Base\ area = b \times h = 6.2 \times 5.3 = 32.86\ cm^2$

$Volume = \frac{1}{3} \times 32.86 \times 8.5$

$Volume = 93.1\ cm^3$



Here the base is a triangle

$Base\ area = \frac{b \times h}{2} = \frac{13 \times 9}{2} = 58.5\ cm^2$

$Volume = \frac{1}{3} \times 58.5 \times 16$

$Volume = 312\ cm^3$

Online clips

M765, M722, M697, U484, U116, U617

Surface Area

Component Knowledge

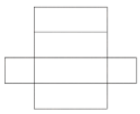
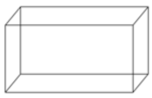
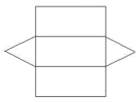
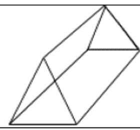
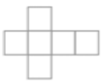
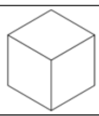
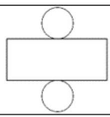
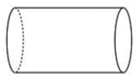
- To be able to calculate the surface area of cuboids, prisms, cones, spheres and composite shapes.

Key Vocabulary

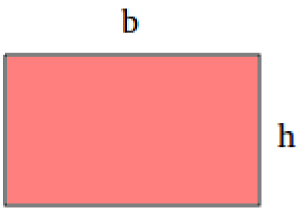
Surface area	The space needed to cover the outside of a 3D shape.
Face	The flat part of a 3D shape.
Cuboid	A 3D object made up of 6 rectangular faces.
Prism	A 3D object in which the two ends are identical.
Cone	A 3D object which tapers from a circular or roughly circular base to a point.
Sphere	A round 3D object.
Composite shape	Is an object made up of two or more other shapes.

Prior knowledge required:

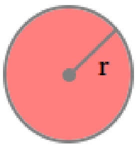
A net of a 3D shape is useful in calculating its surface area. The shape can be unfolded to form a net. This helps us identify the lengths of the sides so we can calculate the area of all the faces. Some common nets are shown below.

Cuboid		6 rectangles		**Triangular prism**		2 triangles 3 rectangles	
Cube		6 squares		**Cylinder**		1 rectangle 2 circles	

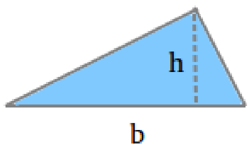
Area formulae which may be useful are shown below



$A = bh$



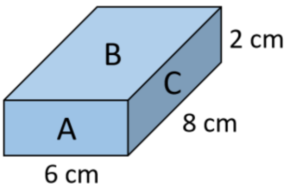
$A = \pi r^2$

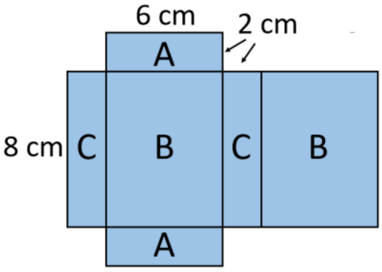


$A = \frac{b \times h}{2}$

Surface Area- cuboids

Find the surface area:





A: b x h

$6 \times 2 = 12cm^2$

B: b x h

$6 \times 8 = 48cm^2$

C: b x h

$2 \times 8 = 16cm^2$

Find the areas of all the faces using the net.

Add all the areas to find the total surface area.

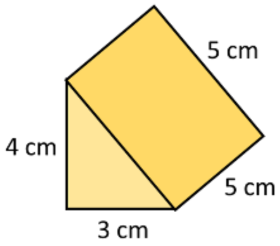
Total surface area

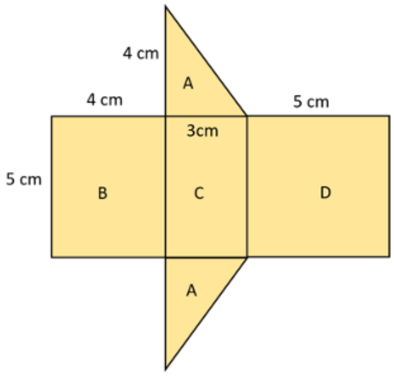
=12+12+48+48+16+16

= $152cm^2$

Surface Area- prisms

Find the surface area:





Working out

A: $\frac{b \times h}{2} = \frac{3 \times 4}{2} = 6cm^2$

B: b x h = 4 x 5 =$20cm^2$

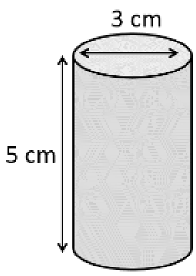
C: b x h = 3 x 5 = $15cm^2$

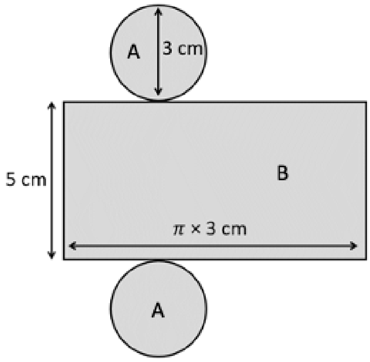
D: b x h = 5 x 5 = $25cm^2$

Total Surface Area = 6+6+20+15+25= $\underline{72cm^2}$

Surface Area- cylinders

Note: the base of the rectangle is equal to the circumference of the circle as it wraps around the curved edge.





Working out

A: $A = \pi r^2 = \pi(3)^2 = 9\pi$

B: b x h = 3π x 5 = 15π

Total Surface Area = $9\pi + 9\pi + 15\pi = 33\pi$

= $103.6725576cm^2$

= $\underline{103.67\ cm^2\ \text{(2dp)}}$

Surface Area- cylinders

Online Clips

Similar shapes

Component Knowledge

- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles in similar shapes
- Understand similarity & congruence

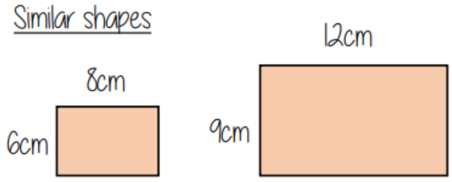
Key Vocabulary

Enlarge	Make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale factor	The multiplier of enlargement
Similar	When one shape can become another through a reflection, rotation, enlargement of translation
Corresponding	Items that appear in the same place in two similar situations

Identifying similar shapes

Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180^0

Similar shapes



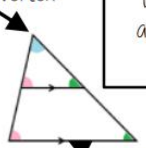
Scale Factor:
Both sides on the bigger shape are 1.5 times bigger

Compare sides: 6 : 9 → **2 : 3** 8 : 12 → **2 : 3**

Both sets of sides are in the same ratio

Similar triangles

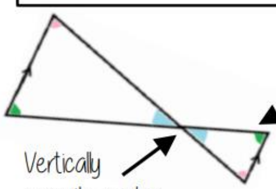
Shares a vertex



Because corresponding angles are equal the highlighted angles are the same size

Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

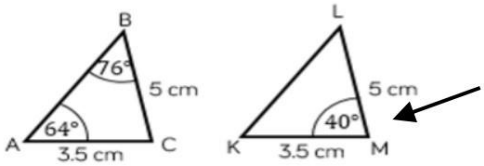


All the angles in both triangles are the same and so similar

Vertically opposite angles

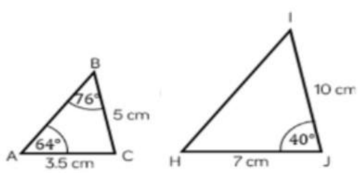
Congruence & similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



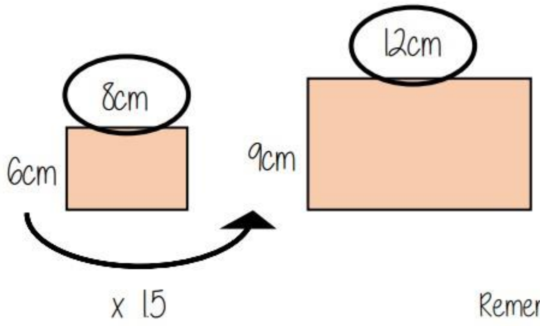
$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Information in similar shapes



Compare the equivalent side on both shapes

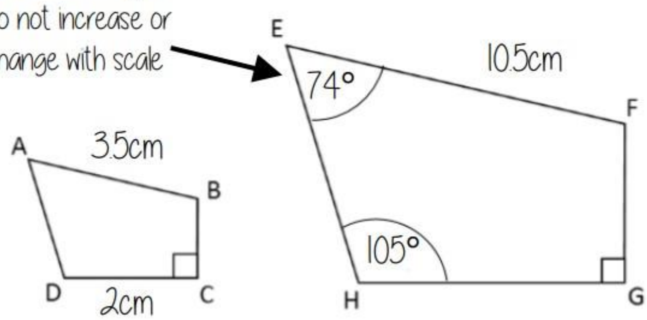
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

AB and EF are corresponding

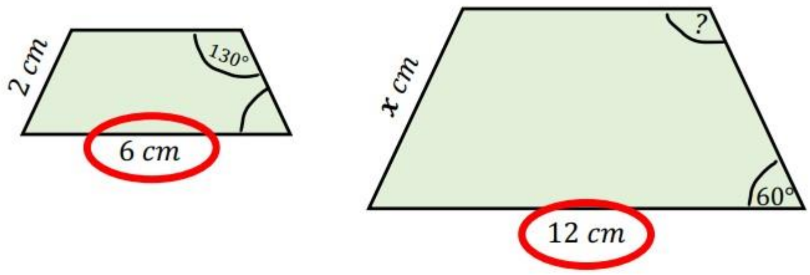
Remember angles do not increase or change with scale



Further example

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



Corresponding sides identify the scale factor $\frac{12}{6} = 2$ Scale Factor = **2**

Calculate the missing side Length (corresponding side) x scale factor

$2cm \times 2$

$x = 4cm$

Enlargement does not change angle size

Calculate the missing angle Corresponding angles remain the same

$130°$

Online clips

M124, M377, M324, M606

Enlargement

Component Knowledge

- Enlarge a rectilinear shape by a given positive scale factor
- Enlarge a rectilinear shape, given a positive integer scale factor and a centre
- Enlarge a rectilinear shape, given a positive fractional scale factor and a centre
- Describe an enlargement in terms of scale factor and centre

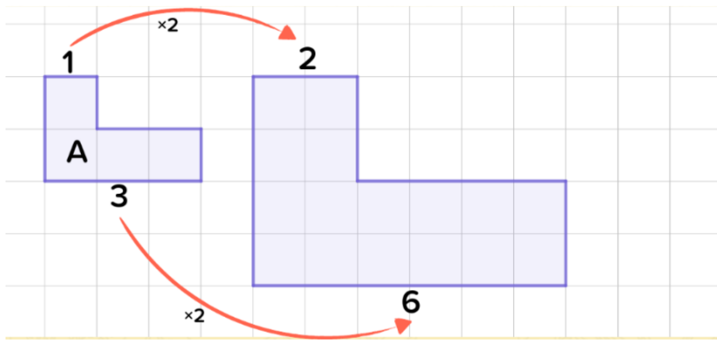
Key Vocabulary

Enlargement	A transformation of a shape in which all dimensions are multiplied by the same number
Scale factor	The number by which dimensions are multiplied in an enlargement
Centre of enlargement	The point from which distances to the *object* and the *image* of an enlargement are measured

Enlarging by a scale factor

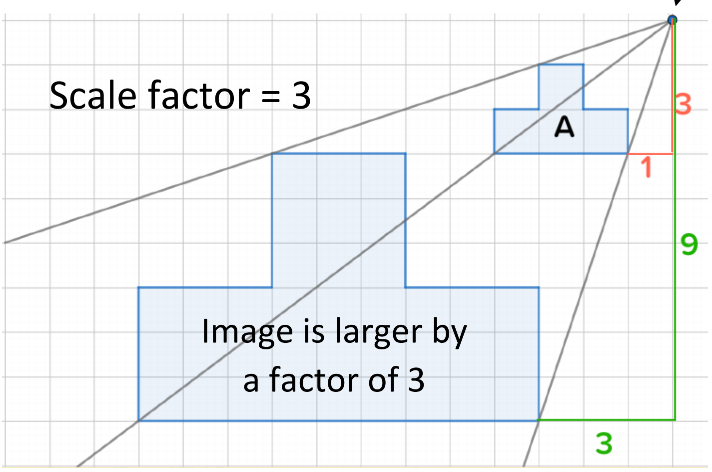
In an *enlargement* all dimensions are multiplied by the same number, called the **scale factor**. In this example shape A has been enlarged by scale factor 2.

If the scale factor is smaller than 1 the dimensions are in fact reduced (divided), although the transformation is still called an enlargement! (See next page)

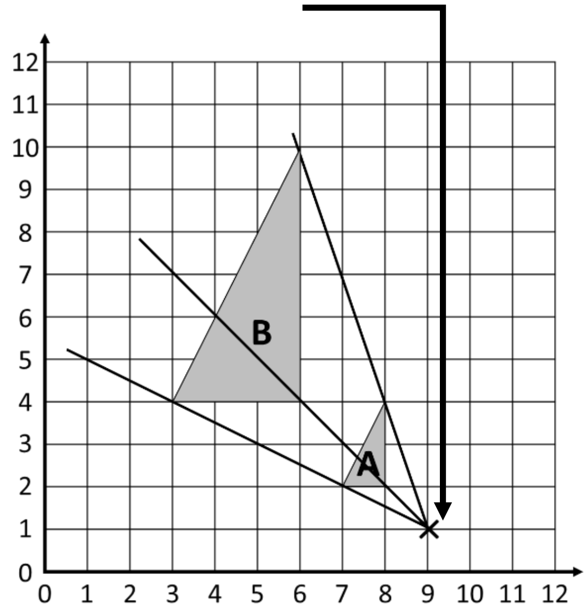


Enlarging by a positive *integer* scale factor from a centre

Centre of enlargement



Measure the distance from the centre of enlargement to each vertex of the *object* shape A; the corresponding vertex in the *image* is triple that distance in the **same** direction

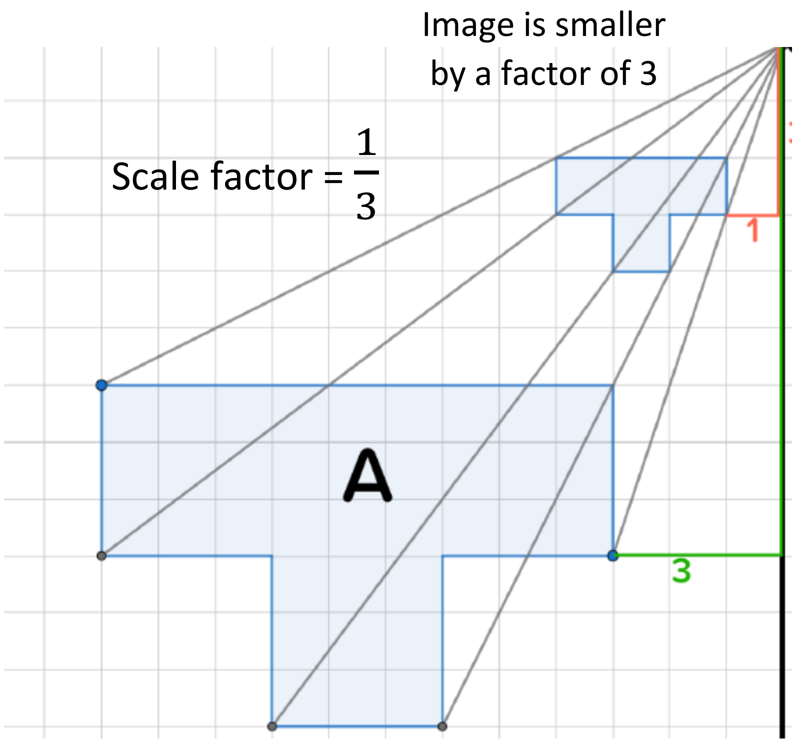


If the object shape is drawn on a coordinate grid, the centre may be specified by coordinates (here the centre is (9,1))

Enlarging by a positive *fractional* scale factor from a centre

A positive scale factor that is smaller than 1 reduces the dimensions of the object shape.

Here the distance from the centre of enlargement to each vertex of the object shape A is measured and then **divided** by 3 to find the corresponding vertex in the image (still in the same direction)



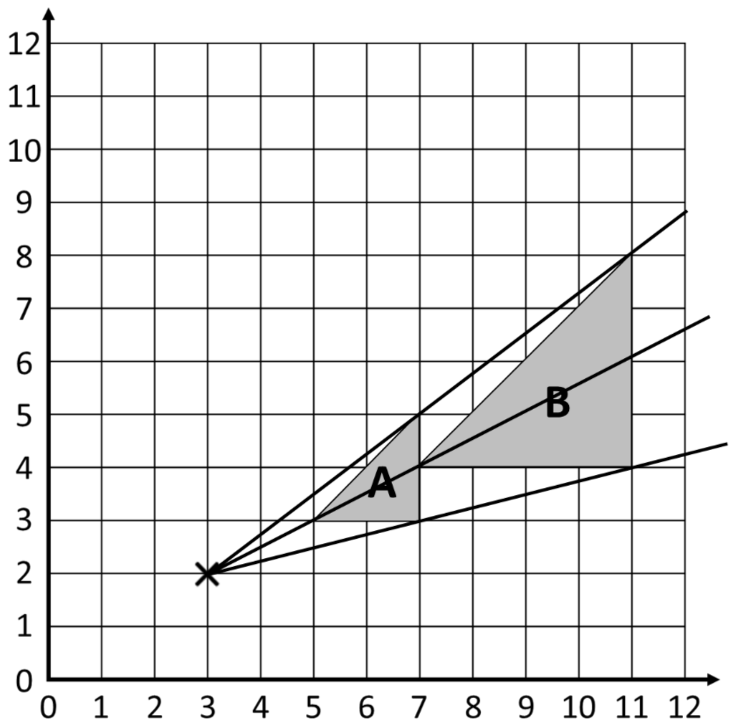
Describing an enlargement

An enlargement is easily identified as such by the change in dimensions.

To determine the scale factor, calculate the ratio of the lengths of corresponding sides in the object and its image.

For the centre, draw lines through two pairs of corresponding vertices and find their point of intersection (thus retracing the steps of the process of enlarging)

The enlargement shown here – from A to B – has scale factor 2 and centre (3,2)



<u>Online clips</u>

M178, U519

Time and Timetables

Component Knowledge

- Be able to add on times to solve worded problems
- Be able to read and interpret different timetables

Key Vocabulary

Time	The ongoing sequence of events taking place
Timetable	A table of information showing when things will happen
Journey	An act of travelling from one place to another
Hour	A period of time equal to a twenty-fourth of a day (1 hour = 60 mins, 1 day = 24 hours)
Second	The basic unit of time. There are 60 seconds in 1 minute and 3600 seconds in an hour
Minute	A unit of time equal to 60 seconds. There are 60 minutes in an hour

Analogue and Digital Clocks

There are **24 hours** in **one day**, but the day can be measured by splitting it into two halves.

The first 12 hours of the day – **from midnight to midday** – are called **AM**, and the next 12 hours are called **PM**.

Each hour has 60 minutes, each minute has 60 seconds.

We use analogue and digital clocks to tell the time.

Analogue clocks show time passing by moving hands. Digital clocks show the time numerically.

Timetables

An important life skill is that we know how to read and understand information offered to us in a variety of different formats and styles eg train and bus timetables.

Bus Timetable					
Thornton Interchange	06:00	06:15	06:30	06:45	07:00
Main Road	06:10	06:25	06:45	07:00	07:10
Crossley Street	06:18	06:33			07:18
Western Road	06:25	06:40	06:57	07:12	07:25
Thornton Drive	06:32	06:47	07:04	07:19	07:32
Saltwell Common	06:40	06:55	07:12	07:27	07:40
Legrams Lane	06:48	07:03			07:48
Thornton Interchange	07:05	07:20	07:28	07:43	08:05

Example

Josh wants to travel from Newcastle to Edinburgh. He wants to leave close to 1pm. Which train will he catch and how long will the journey take?

Josh will catch the 12.54 train from Newcastle which arrives at 14.21

The journey will take 1 hour and 27 minutes

Some train times between Newcastle and Edinburgh	
Leaves Newcastle	Arrives Edinburgh
12:39	14:13
12:54	14:21
13:35	15:09
13:45	15:16
13:52	15:19
14:21	15:47
14:43	16:15
14:55	16:22

Online clips

Q283, Q547, Q291, Q760, Q303, Q493

Measures

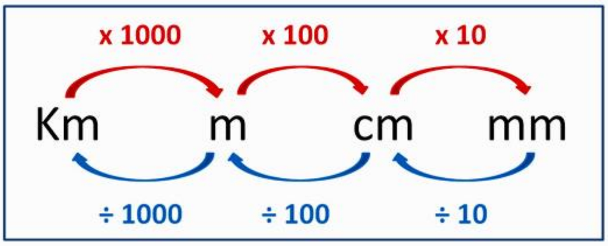
Component Knowledge

- Convert between units of length
- Convert between units of capacity
- Convert between units of mass
- Convert between units of time

Key Vocabulary

Convert	To change from one unit to another such as from centimetres to millimetres, or litres to millilitres, etc.
Unit	A quantity used as a standard of measurement
Length	The measurement of something from end to end
Capacity	The maximum amount that something can contain
Mass	The weight of an object
Time	*A numerical quantity that represents the duration between two events*.

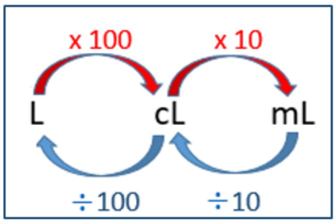
Units of length



5km = ? m Need to x 1000 5 x 1000 = 5000m ✓

120cm = ? m Need to ÷ 100 120 ÷ 100 = 1.2m ✓

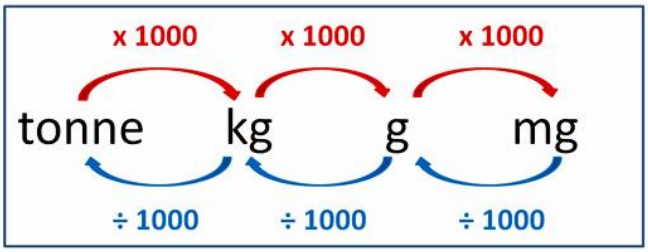
Units of capacity



5L = ? cL Need to × 100 5 × 100 = 500 cL

750 mL = ? cL Need to ÷ 10 750 ÷ 10 = 75 cL

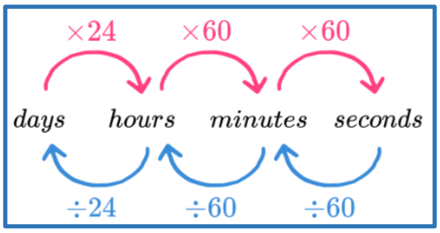
Units of mass



Mass conversions use 1000's, and usually create fairly large results.

1.6 tonne = ? kg Need to x 1000 1.6 x 1000 = 1600 kg ✓

Units of time



2 mins = ? secs need to × 60 2 × 60 = 120 secs

96 hrs = ? days need to ÷ 24 96 ÷ 24 = 4 days

Online clips

M772, M761, M530, M774, M627, M515

Compound units of measure

Component Knowledge

- Be able to convert compound units of measure
- Calculate speed, density and pressure

Key Vocabulary

Speed	How fast something is moving. Measured as distance travelled per unit of time
Density	A measure of how much matter is in a certain volume
Pressure	The force per unit of area
Measure	To find a number that shows the size or amount of something
Convert	To change a value or expression from one form to another

Speed, density and pressure are examples of compound measures which means they are made up of two or more other measures. For example, speed is measured using distance and time (mph, m/s etc)

To convert the units of compound measures, convert the individual units separately

Useful conversions to know

1cm	10mm
1m	100cm
1km	1000m
1g	1000mg
1kg	1000g
1 hour	3600 secs
1 hour	60 mins
1 min	60 secs

Example

The maximum speed of a racing car is 340 km/h. Convert this speed into m/s (give your answer to one decimal place)

First convert kilometres into metres

1km – 1000m

340 x 1000 = 340000m

Next convert hours into seconds

1h = 3600 s

Finally combine the two unit conversions

$$340\ km/h = \frac{340000}{3600} m/s$$

= 94.4 m/s

Example

Convert 19.3 g/cm^3 to kg/m^3

$1m^3 = 1000000cm^3$ so 19.3 g/cm^3 = 19300000 g/m^3

1000g = 1kg so 19300000 g/m^3 = 19300 kg/m^3

Online clips

M627, M515, M774

Area of 2-D shapes

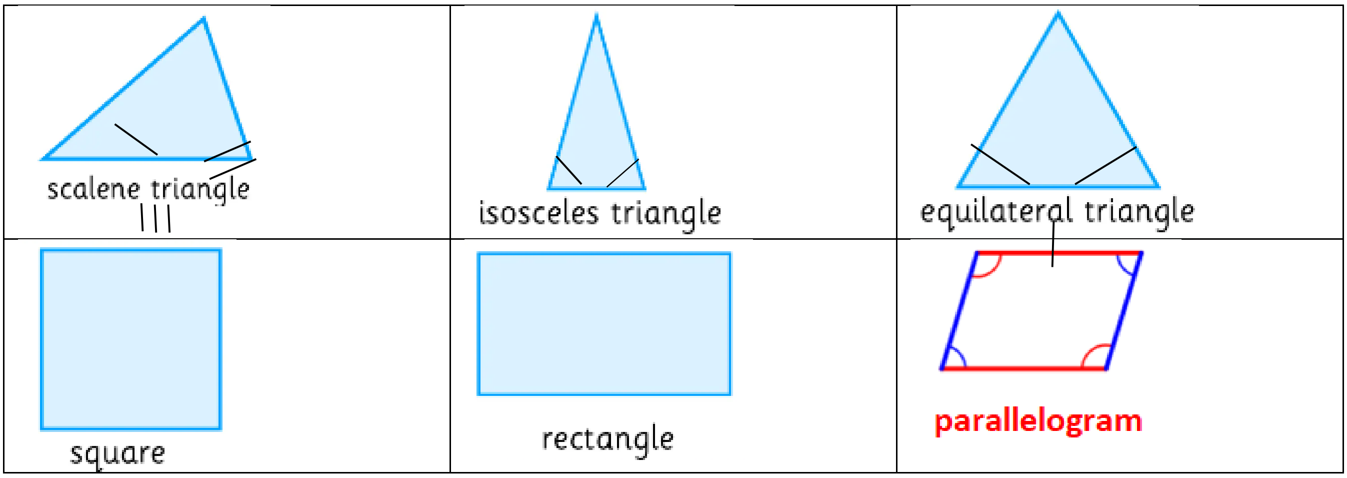
Component Knowledge

- Identify the relevant dimensions
- Identify the correct formula for area
- Express the answer in the correct units

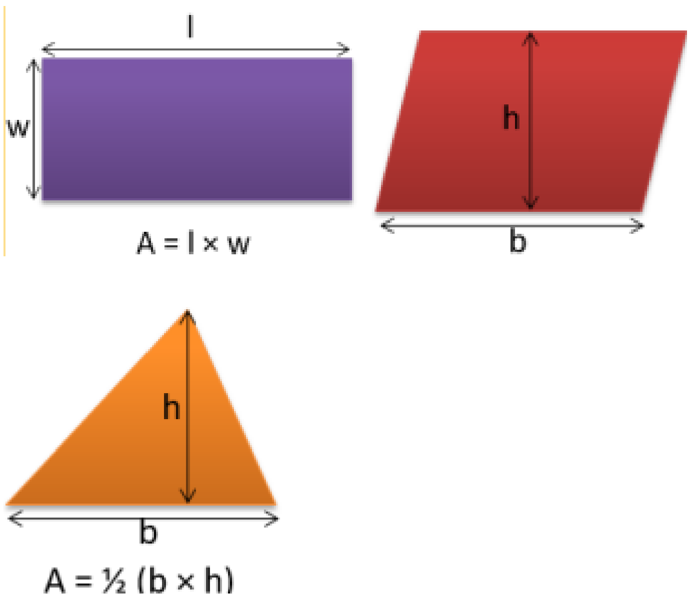
Key Vocabulary

Area	The amount of squared units that fit inside a shape
Dimension	The lengths of the sides of the shape
Unit of measure	This can be length (cm, mm, m) or area (cm^2, mm^2)
Compound shape	A 2-D shape composed of key 2-D shapes

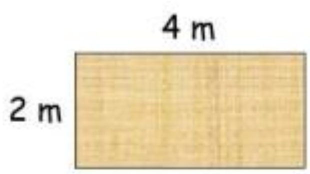
2-D Shapes



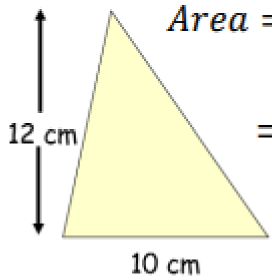
Area Formulae



Examples



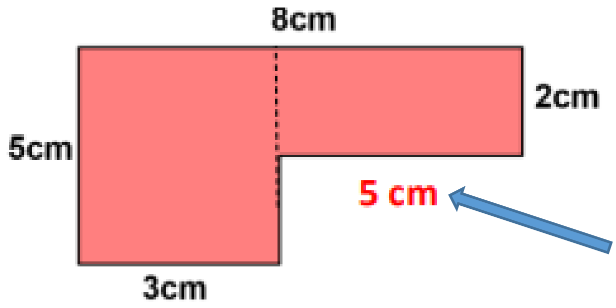
$$Area = 2 \times 4 = 8m^2$$



$$Area = \frac{1}{2}(10 \times 12)$$

$$= 60cm^2$$

Compound shape example

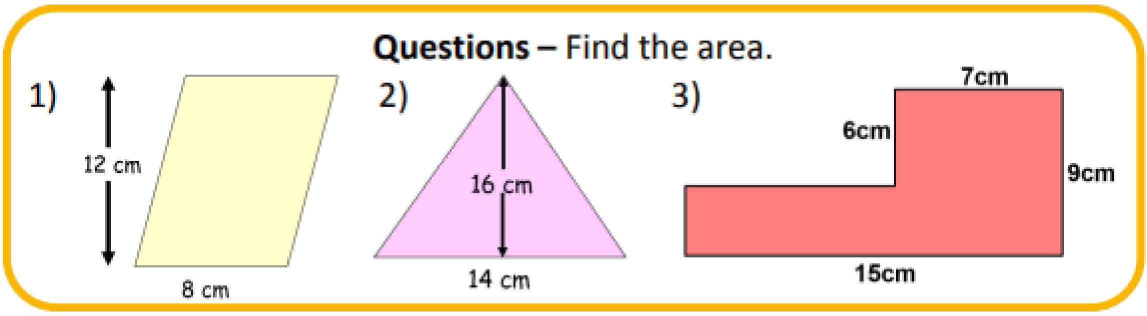


You must determine any missing dimensions,

e.g. 8 – 3 = 5cm

$$Area = (5 \times 3) + (2 \times 5)$$
$$= 25cm^2$$

Further examples



1)

$$Area = 12 \times 8 = 96cm^2$$

2)

$$Area = \frac{14 \times 16}{2} = 112cm^2$$

3)

$$Area = (6 \times 7) + (15 \times 3) = 87cm^2$$

Online clips

M900, M390, M291, M610, M269, M996