Pie charts

Component Knowledge

- Calculate angles in a pie chart
- Draw a pie chart from a table
- Interpret pie charts using fractions
- Interpret pie charts using angles

Key Vocabulary

Angle	The amount of turn between 2 lines.
Pie chart	A chart that displays data proportionally.
Protractor	Equipment used to measure and draw angles

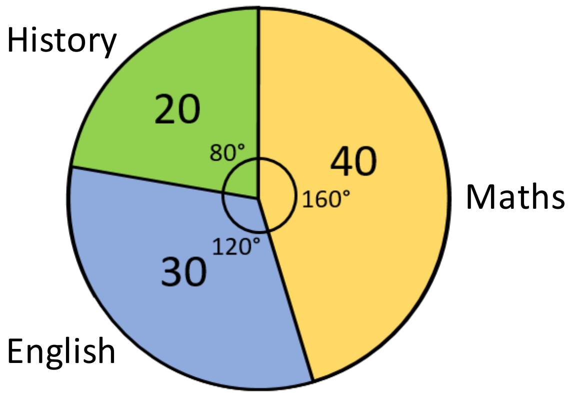
Drawing pie charts

How many degrees for one person? $\frac{360}{90} = 4°$

360 ÷ total = degrees for one person. In this example one person is 4°.

Subject	Number of Students	Calculation	Angle
Maths	40	40 × 4°	160°
English	30	30 × 4°	120°
History	20	20 × 4°	80°
Total	90		360°

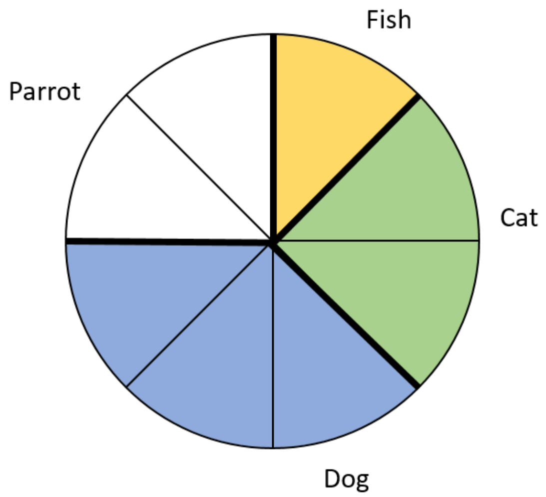
Multiply number of students by 4° to get the angle.



Draw the angles onto the pie chart. Label each part with what it is (subject in this example) and how many it represents (40 for Maths in this example).

Interpret pie charts (fractions)

A class of **32 students** were surveyed to find their **favourite pet**.
The **pie chart** shows the total answers. How popular was each animal?



The pie chart is split into 8 pieces, so each sector is worth $\frac{1}{8}$ of 32 = 4

Fish: $\frac{1}{8}$ of 32 = 4

Cat: $\frac{2}{8}$ of 32 = 8

Dog: $\frac{3}{8}$ of 32 = 12

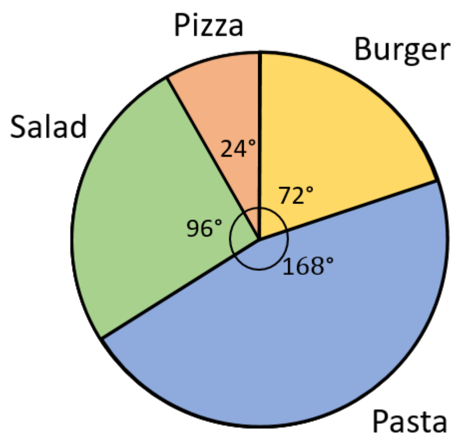
Parrot: $\frac{2}{8}$ of 32 = 8

Check that the totals add up to the original total in the question.
(4 + 8 + 12 + 8 = 32)

Interpret pie charts (angles)

150 students were surveyed about their favourite food.

Favourite Food	Angle	Calculation	Frequency
Burger	72°	$\frac{72}{360} \times 150$	30
Pasta	168°	$\frac{168}{360} \times 150$	70
Salad	96°	$\frac{96}{360} \times 150$	40
Pizza	24°	$\frac{24}{360} \times 150$	10



To calculate the frequency from a pie chart when you are given the angle, you do the opposite of what you do to calculate the angle.

Angle ÷ 360 × total frequency

Online clips

M574, M165

Systematic listing and Product rule

Component Knowledge

- To be able to list the possible outcomes of different events.
- To be able to use the product rule to determine the number of outcomes for different events.

Key Vocabulary

Outcome	The possible result of an experiment
Product	The answer when two or more numbers are multiplied together.

Systematic listing

Systematic listing is the method of listing all the possible outcomes of an event.

Worked example

At the ice cream kiosk, you can choose…**one flavour** of ice cream and **one topping**.

Flavour	Toppings
Vanilla	Flake
Chocolate	Sprinkles
Banana	Nuts

There are 9 possible combinations:
Vanilla and Flake, Vanilla and Sprinkles, Vanilla and Nuts
Chocolate and Flake, Chocolate and Sprinkles, Chocolate and Nuts
Banana and Flake, Banana and Sprinkles, Banana and Nuts

Product rule for counting

Product rule uses multiplication to determine the number of possible outcomes of an event rather than listing them all.

Worked example.

A safe has a 4-digit combination for example 4 5 7 8

Use the product rule to find the number of 4-digit combinations you can have on this safe.

Each digit has a possible 10 possibilities (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9)

Number of combinations = 10 (1st digit) x 10 (2nd digit) x 10 (3rd digit) x 10 (4th digit) = 10,000

Online clip

U369

Probability

Component Knowledge

- Understand what probability shows
- Understand probability notation
- Write a probability of a single event

Key Vocabulary

Probability	The mathematical chance, likelihood, of an outcome happening
Event	The "thing" that is being completed/done/observed/counted
(Event) Outcome	What happens when the event is performed
Probability scale	A numerical scale from 0 to 1, with 0 being an impossible outcome and 1 being an outcome certain to happen
Mutually exclusive (event) outcomes	When outcomes cannot happen at the same time eg being an adult and being a child, you **cannot be both**
Exhaustive (event) outcomes	When a set of outcome cover all possibility with no gaps eg it snowing and it not raining

Probability:

The probability of an (event) outcome A, happening is

$$P(outcome\ A) = \frac{number\ of\ ways\ outcome\ A\ can\ happen}{number\ of\ ways\ any\ outcome\ can\ happen}$$

e.g. the probability of rolling a number 4 on a regular 6 sided dice

$Outcome\ "4": \mathbf{4},\ so\ \ \mathbf{1\ option}$

$$P(roll\ a\ 4) = \frac{1}{6}$$

$All\ possible\ outcomes: \mathbf{1, 2, 3, 4, 5}\ or\ \mathbf{6},\ so\ \ \mathbf{6\ possibilities\ altogther}$

e.g. the probability of rolling a number greater than 4 on a regular 6 sided dice

$Outcomes\ "greater\ than\ 4": \mathbf{5}\ or\ \mathbf{6},\ so\ \ \mathbf{2\ options}$

$$P(roll\ a\ number\ greater\ than\ 4) = \frac{2}{6}$$

$All\ possible\ outcomes: \mathbf{1, 2, 3, 4, 5}\ or\ \mathbf{6},\ so\ \ \mathbf{6\ possibilities\ altogther}$

Online clips

M655, M941, M938, M755

Frequency trees

Component Knowledge

- Complete a frequency tree from given information.
- Calculate probabilities from a frequency tree

Key Vocabulary

Frequency	The number of times an event occurs.
Probability	The chance that something will happen.
Frequency tree	Used to record and organise the frequency of events occurring.

Frequency trees are a way of organising information. They can be used to solve probability problems.

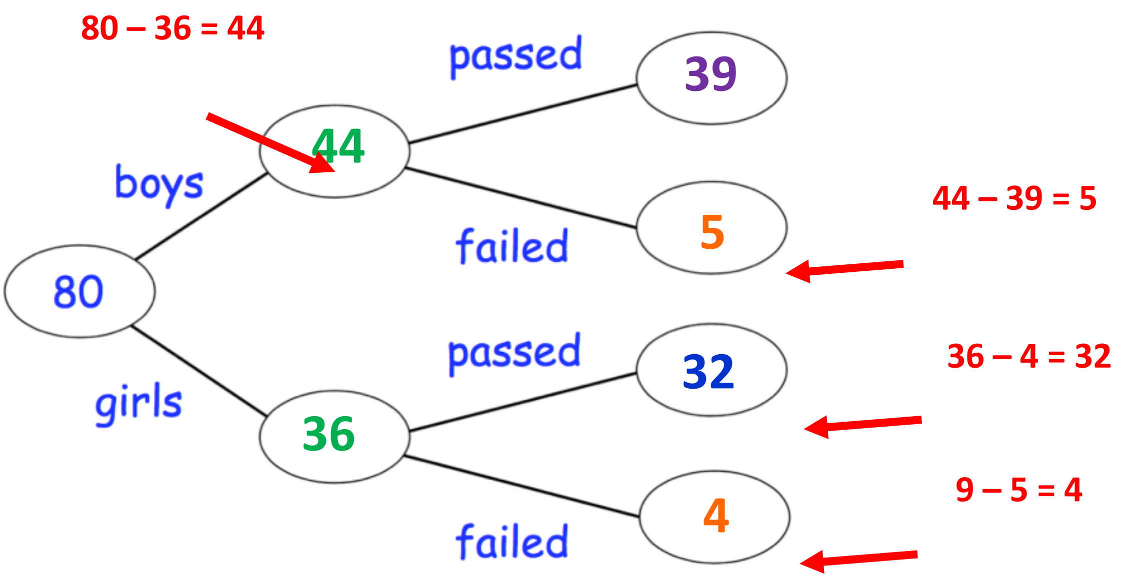
We start with the total number of items and then divide these items into two or more categories, writing down the frequency of items in each category.

A group of 80 boys and girls sat a test.

36 of the children are girls.

9 of the 80 children failed the test.

39 of the boys passed the test.



One of the boys is chosen at random.

Work out the probability that the boy failed the test. $\frac{5}{44}$

Number of boys who failed.

Total number of boys.

Online clip

U280

Relative Frequency

Component Knowledge

- Understand what relative frequency is
- Calculate experimental probability
- Use relative frequencies or experimental probabilities to estimate expected outcomes

Key Vocabulary

Probability	The mathematical chance, likelihood, of an outcome happening
Event	The "thing" that is being completed/done/observed/counted
(Event) Outcome	What happens when the event is performed
Probability scale	A numerical scale from 0 to 1, with 0 being an impossible outcome and 1 being an outcome certain to happen
Theoretical probability	Probability based on reasoning
Experimental Probability	Probability estimated from the results of conducting an experiment (set of observations)
Frequency	The number of times something happens
Relative frequency	The number of times an event outcome happens relative to the number of times the event takes place (number of times experiment is conducted)
Number of Trials	The number of times an experiment is conducted
Expected outcomes (Expectation)	The number of times you would expect a particular (event) outcome to happen for a specified number of trials

Experimental Probability:

An ***estimate*** of the probability of an (event) outcome A, happening when an experimental is conducted

$$ExP(outcome\ A) = \frac{number\ times\ outcome\ A\ happened}{number\ of\ times\ event\ takes\ place\ (total\ number\ of\ trials)}$$

e.g. If a *biased* coin is flipped 20 times and lands on tails 7 times

$$ExP(lands\ on\ \boldsymbol{tails}) = \frac{7}{20}$$

Relative Frequency:

The number of times (frequency) an (event) outcome A happens, in relation to the number of times the event is performed

$$Rf\ (outcome\ A) = \frac{number\ times\ outcome\ A\ happened}{number\ of\ times\ event\ takes\ place\ (total\ number\ of\ trials)}$$

e.g. If a *biased* coin is flipped 20 times and lands on tails 7 times

$$Rf(lands\ on\ \boldsymbol{tails}) = \frac{7}{20}$$

Relative frequencies are commonly written as decimal $Rf(lands\ on\ \boldsymbol{tails}) = 0.35$

Relative Frequency v Experimental Probability:

Can be thought of as inter-changeable, relative frequency is used as an experimental probability.

Expectation:

Relative frequency can be used to estimate the probability of an (event) outcome A happening, and hence the expected number of times (event) outcome A would happen over a given number of observations (experiments)

$$Expectation\ of\ outcome\ A\ = Rf(A) \times number\ of\ trials$$

Eg The results of rolling a biased 6-sided dice 30 times are recorded in the table

Score	1	2	3	4	5	6
Frequency	4	5	2	8	4	7

The relative frequencies can be calculated by $\frac{frequency}{total\ number\ of\ trials\ (rolls\ of\ dice)}$

Rel Freq	$\frac{4}{30}$	$\frac{5}{30}$	$\frac{2}{30}$	$\frac{8}{30}$	$\frac{4}{30}$	$\frac{7}{30}$

i. ***Estimate*** the number of times the dice would land on 4, if rolled 120 times

$$Expectation\ of\ \text{"}lands\ on\ 4\text{"}\ = Rf(lands\ on\ 4) \times number\ of\ trials$$

$$Expectation\ of\ \text{"}lands\ on\ 4\text{"} = \frac{8}{30} \times 120$$

$$= \frac{8 \times 120}{30}$$

$$= 32$$

When rolled 120 times we would expect the dice to land on a 4, 32 times.

Note: Like Probabilities, relative frequencies should always sum to 1.

<u>Online clips</u>

M332, M206

Sample Spaces

Component Knowledge

- Complete a sample space diagram to show possible outcomes
- Calculate probabilities from a sample space diagram

Key Vocabulary

Outcome	The way something turns out
Sample space	Records the possible outcomes of two different events happening
Event	A thing that happens or takes place
Probability	The chance of an event happening
Independent	Events which do not have an effect on each other
Dependent	Has an effect on something else – eg Not replacing a counter when taking multiple out of a bag

Creating a sample space diagram

1. **Use information proved to decide whether to write a list or create a table to find all possible outcomes.**
2. **Systematically write the list or fill in the table by either listing outcomes or performing operations with values.**
3. **Use the information from the list or table to find any probabilities required.**

This is what a sample space would look like for spinning a spinner and flipping a coin

		Spinner		
Coin		Red	Green	Blue
	Heads	H,R	H,G	H,B
	Tails	T,R	T,G	T,B

Finding a probability from a sample space

Two dice are thrown and the possible outcomes are shown in the sample space diagram below:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

1) What is the probability that 2 numbers which are the same are rolled?

$$\frac{6}{36} = \frac{\textit{outcomes where numbers are the same}}{\textit{total number of outcomes}}$$

2) What is the probability that two even numbers are rolled?

$$\frac{9}{36} = \frac{\textit{outcomes where numbers are both even}}{\textit{total number of outcomes}}$$

Creating a table helps to organise the information you have and ensures that no outcomes are missed or repeated.

You might also be asked to do a calculation to fill in the sample space instead of just putting the outcomes straight in.

This sample space shows the difference between the outcomes when 2 dice are rolled.

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Online clip

M718

Two-way Tables

Component Knowledge

- Construct two-way tables.
- Read and interpret two-way tables.
- Find probabilities using two-way tables.

Key Vocabulary

Two-way table	A table which shows two variables at the same time- we can read them vertically and horizontally.
Horizontal	Reading from left to right or right to left
Vertical	Reading the table top to bottom or bottom to top
Variable	A way of organising data according to a shared characteristic e.g eye colour, age

We use two-way tables to compare 2 variables

To construct a two-way table, we need two variables. One variable is featured as the top row within the two-way table (read horizontally), and the other variable features on the first column of the table (read vertically).

Example This two way table shows a data set about what students eat for lunch.

	Boys	Girls	Total
Cooked food	18	22	40
Packed lunch	17	33	50
Total	35	55	90

The first column shows the type of food chosen.

The top row shows boy or girl.

17 boys had a packed lunch

90 students were asked in total (40+50=90 and 35+55=90

Example: 80 children went on a school trip.

They went to London or to York.

23 boys and 19 girls went to London.

14 boys went to York.

(a) Use this information to complete the two-way table.

	London	York	Total
Boys	23	14	
Girls	19		
Total			80

Step 1- fill in all known values from the question.

Total = 80

Boys in London = 23

Girls in London = 19

Boys in York = 14

Example: 80 children went on a school trip.

They went to London or to York.

23 boys and 19 girls went to London.

14 boys went to York.

(a) Use this information to complete the two-way table.

Step 2- calculate missing values using the known values. Remember both the horizontal and vertical totals must equal the overall total, in the case below, = 80.

	London	York	Total
Boys	23	14	37
Girls	19	24	43
Total	42	38	80

23 + 19 = 42
Boys total

80 – 37 = 43
Girls total

23 + 19 = 42
London total

80 – 42 =38
York total

38 – 14 = 24
Girls in York

Interpreting two-way tables

We can now use the fully completed two-way table to interpret the data.

	London	York	Total
Boys	23	14	37
Girls	19	24	43
Total	42	38	80

Questions could look like this:

a) How many students went to London?

We can read from the table vertically and see there **were 42 students who visited**

b) One of these 80 students is chosen at random.
What is the probability that this student visited London?

We can read from the table vertically and see there **were 42 students who visited London.**

So, the $P(a\ student\ visits\ London) = \frac{42}{80}$

c) A student is picked at random.
Given they are a girl, what is the probability they went to York?

We can read the table to find the **total girls = 43** and the **girls who visited York = 24**

So, the $P(given\ the\ student\ is\ a\ girl,\ they\ visit\ York) = \frac{24}{43}$

Online clip M899

Venn Diagrams

Component Knowledge

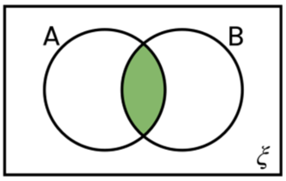
- Complete a Venn Diagram when given a set of data
- Fill in missing values in a Venn Diagram
- Interpret a Venn diagram
- Find probabilities from a Venn Diagram
- Use simple set notation

Key Vocabulary

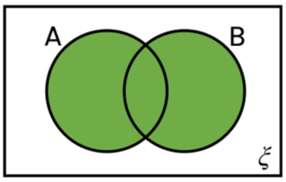
Set	A collection of "things" (objects or numbers)
Union	The set made by combining the elements of two sets
Intersection	The intersection of two sets has only elements common to both sets
Probability	The change that something happens
Venn Diagram	A diagram that shows sets which elements belong to which set by drawing regions around them. It is used to represent data that has an overlap.

Key Concepts

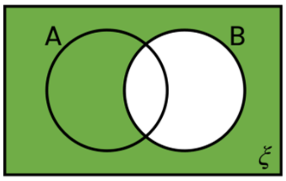
Venn diagrams show all possible relationships between different sets of data.



A ∩ B
The **intersect** of A and B.
The set of elements in **both A and B**.



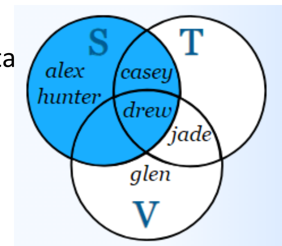
A ∪ B
The **union** of A and B.
The set of elements in **A or B or both.**



B'
The **complement** of B.
The set of elements **not in B**.

Venn Diagrams with 3 sets

Diagrams can be drawn to show more than 2 sets of data This is an example of a Venn Diagram containing 3 sets.



S = {Alex, Hunter, Casey and Drew}

T = {Jade, Casey and Drew}

V = {Drew, Jade and Glen}

Example

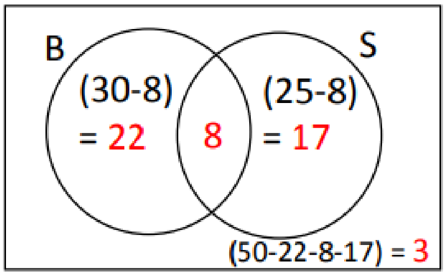
Out of 50 people surveyed:

30 have a brother

25 have a sister

8 have both a brother and a sister

This is what the Venn Diagram for this information would look like



Remember – the people in the intersection are also included in the whole circle so we don't duplicate data.

From the Venn Diagram, we can see that the probability of someone from this group just having a brother is 22/50.

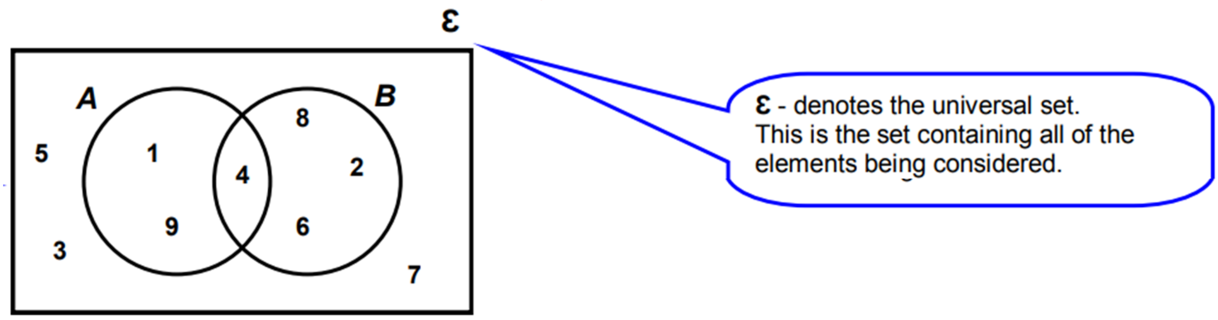
The probability of someone from this group having neither a brother or a sister is 3/50.

The probability of having a brother and a sister,

$$P(A \cap B) = \frac{8}{50}$$

Example: Given a set of numbers

Ɛ={1,2,3,4,5,6,7,8,9}
A={square numbers}
B={even numbers}



In set A 'the square numbers' are 1, 4 and 9.

In set B the 'even numbers' are 2, 4, 6, 8.

4 is in both groups so would go in the centre (the intersection)

Outside of the circles are any numbers remaining in ε

Online clips

M829, M419, M834

Averages

Component Knowledge

- To understand and calculate the mode from a list.
- To understand and calculate the median from a list.
- To understand and calculate the mean from a list
- To calculate the range and understand it is **not** an average.

Key Vocabulary

Data set	Collection of values that share a common relationship. This could be answers to a set question or information for a set objective.
Average	Is a value (or values) that is used to represent a whole data set
Mode	The most frequent value in a data set. It is a type of average. Modal is another word used more mode.
Median	The middle value of a data set, when ordered. It is a type of average.
Mean	A measure of the size of the data when shared out equally. It is a type of average.
Range	A value to show spread out a data set is. It can be used to describe how representative of the whole data set the average used is. IT IS NOT AN AVERAGE.

Averages

We use averages to summarise a whole data set in a single value/few values. We do this so we can interpret large data sets and also compare data sets more easily.

Mode- the most frequent value/ few values in a data set. There can also be no mode in a set of data.

Ex 1, find the mode:

blue red blue green blue blue
pink green blue red blue yellow

Blue is the mode.

Ex 2, find the mode:

9, 4, 3, 6, 9, 5, 2, 1, 8, 7

To make it easier, we can re-write these values in ascending(increasing) order.

1, 2, 3, 4, 5, 6, 7, 8, 9, 9. We can now see clearly **9 is the mode**.

Ex 3, find the mode:

9, 4, 3, 6, 9, 5, 2, 1, 8, 7, 3

Re-written 1, 2, 3, 3, 4, 5, 6, 7, 8, 9, 9 We can see **3 and 9 are the modal values**.

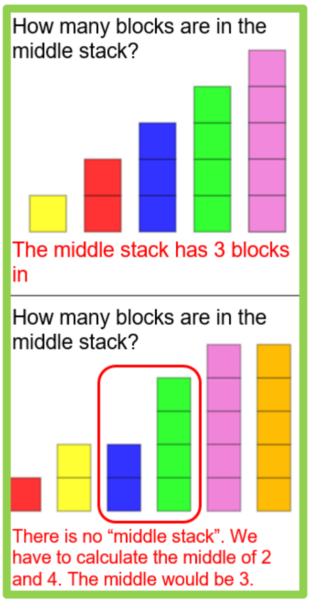
** We usually only have 1, 2 or 3 modal values**

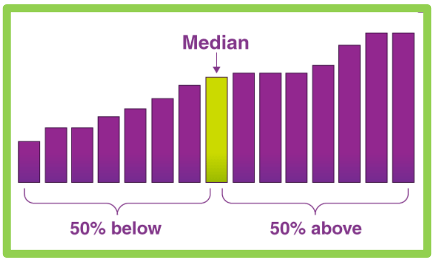
Ex 4, find the mode:

4, 3, 6, 9, 5, 2, 1, 8, 7

Re-written 1, 2, 3, 4, 5, 6, 7, 8, 9 We can see there **are NO modal values**.

Median- the middle value in a data set, when in order. If there are 2 middle values, we find the midpoint between them.





Find the median of: 1, 3, 3, **6**, 7, 8, 9

Median = <u>6</u>

Find the median of: 1, 2, 3, **4**, **5**, 6, 8, 9

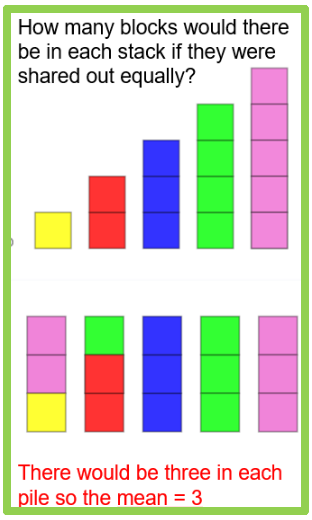
Median is the midpoint of 4 and 5 = <u>4.5</u>

Find the median of the following set of numbers.

40 –2 10 40 –31 3 –34 –13 –10 1 30 16 –16

–34 –31 –16 –13 –10 –2 1 3 10 16 30 40 40

Mean- The mean is the size of each part when a quantity is shared equally. We can do this by adding all the values in the data set together and then dividing it equally between the number of values.



Example 1.
Find the mean of the following set of numbers.

19, 6, 17, 6

Solution.
To find the mean divide the sum of the numbers by the number of numbers.

$$\frac{\text{Sum of numbers}}{\text{Number of numbers}} = \frac{19+6+17+6}{4}$$
$$= \frac{48}{4}$$
$$= 12$$

There are 4 values in the data set so we are dividing by 4.

Range- the range shows how spread out the data is. It is useful to order the data when finding the range. The smaller the range, the more consistent the data.

E.g. Find the range of the following numbers

43 36 10 –8 –3 –6 –4 –22

–22 –8 –6 –4 –3 10 36 43

65

Range =43- -22 = <u>65</u>

Online Clips

M841, M934, M940, M328

Set Notation

Component Knowledge

- Complete a Venn Diagram when given a set of data
- Fill in missing values in a Venn Diagram
- Find probabilities from a Venn Diagram

Key Vocabulary

Set	A collection of "things" (objects or numbers)
Union	The set made by combining the elements of two sets
Intersection	The intersection of two sets has only elements common to both sets
Complement	All elements from a universal set not in our set
Element	Things contained in a set

Key Concepts

A set can be a list of items known as elements

A subset would be a selection of these elements.

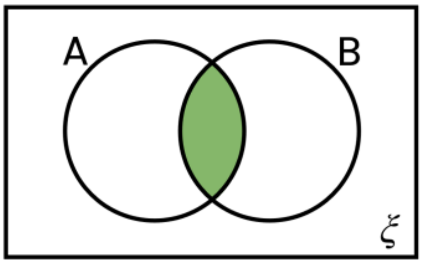
When we list elements within a set, we use these curly brackets { } and separate each elements in the list with commas.

The universal set, ξ, is the list of every element that there is available to choose from.

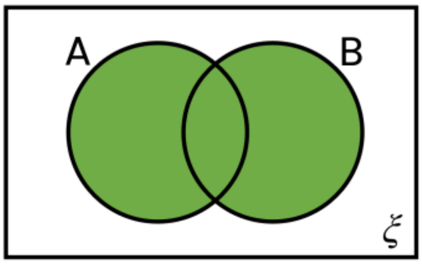
The complement of a set is denoted with an apostrophe and would be the remaining elements in the universal set that are not part of that set.

Symbol	Description
$\{\}$	Curly brackets - contain all items in a set
,	Comma - separates items in a set
'	Complement - the items not in a set
ξ	The Universal Set - contains all items in every set and subset required
ϕ	The Empty Set - contains no items
A	Set A
A'	Not Set A (the complement of Set A)
B	Set B
B'	Not Set B (the complement of Set B)
$A \cap B$	A and B (A intersection B)
$(A \cap B)'$	Not A and B (the complement of A intersection B)
$A \cup B$	A or B (A union B)
$(A \cup B)'$	Not A or B (the complement of A union B)
$n(A)$	The number of elements in A. The cardinality of A

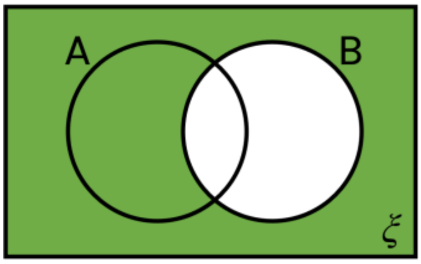
These are the different symbols you may see when working with set notation



A ∩ B
The **intersect** of A and B.
The set of elements in
both A and B.



A ∪ B
The **union** of A and B.
The set of elements in
A or B or both.



B'
The **complement** of B.
The set of elements
not in B.

The shaded sections of the Venn Diagrams show which elements would be included for an intersection, a union or a complement

Example

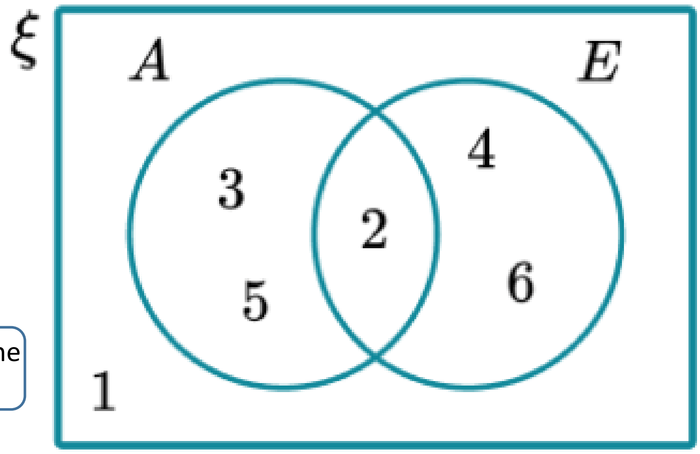
ξ = {1, 2, 3, 4, 5, 6}

The universal set shows us all the elements in the set

A = {2, 3, 5}

E = {2, 4, 5}

Sets A and E are subsets of the universal set



The **complement** of A (not A) is **A'** = {1, 4, 6)

The **union** of A and E (A or E) is **A U E** = {2, 3, 4, 5, 6}

The **intersection** of A and E (A and E) is **A ∩ E** = {2}

Online clips