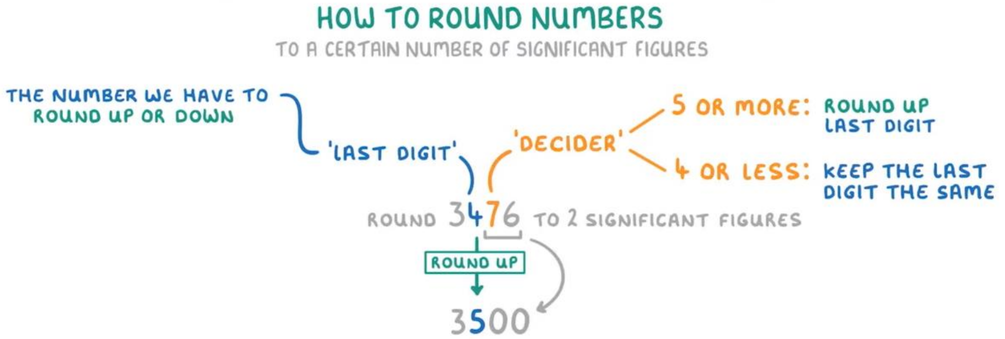
Rounding

What do I need to be able to do?

- To round numbers to the nearest 10, 100, 1000 etc.
- To round numbers to nearest 1, 2 & 3 decimal places
- Round numbers to the nearest 1 significant figure.
- Round numbers to the nearest 2 or 3 significant figures

Key Vocabulary

Round	Making a number simpler but keeping its value close to what it was. The result is less accurate
Significant	The number of digits that are meaningful; they have an accuracy matching our measurements
Integer	A number with no fractional part
Decimal	Based on 10. Decimal number is often used to mean a number that uses a decimal point
Lower Bound	A value that is less than or equal to every element of a set of data
Upper Bound	A value that is greater than or equal to every element of a set of data



Round to	Circle, Underline, Decide	Answer
Nearest 1000	5 7 8 3 . 1 9 9	≈ 6000
Nearest 100	5 7 8 3 . 1 9 9	≈ 5800
Nearest 10	5 7 8 3 . 1 9 9	≈ 5780
Nearest integer	5 7 8 3 . 1 9 9	≈ 5783
1 d.p	5 7 8 3 . 1 9 9	≈ 5783.2
2 d.p	5 7 8 3 . 1 9 9	≈ 5783.20

Rounding decimal places

1. Identify the position of the decimal place to be rounded to, e.g. 2 d.p. would be the 2nd digit after the decimal place.
2. Then look to the right of this digit, this is called the decider, this number now decides whether the decimal place is rounded up or kept the same.
3. If the decider is 5 or more then round the digit up
4. If the decider is 4 or less, then leave the digit as it is.

Significant Figures

The first significant figure is the first non-0 digit, as you read from left to right.

Example 1

Round 3786 to one significant figure

Th H T U

3 7 8 6

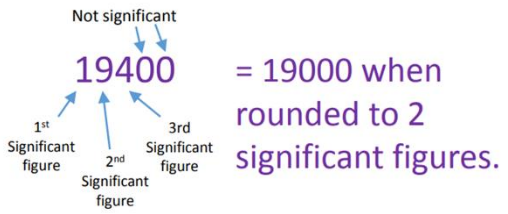
The first significant figure is in the thousand's column so to the nearest thousand it is 4000

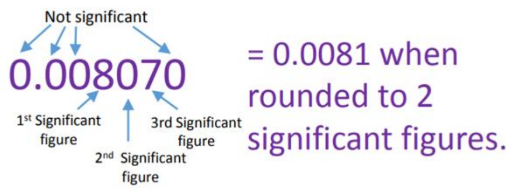
Example 2

Round 0.0145 to two significant figures

0 . 0 1 4 5

The first significant figure is in the thousandth's column, the 5 rounds the 4 up so to the nearest thousandth it is 0.015.





Online clips

M111, M431, M994, M131

Error Intervals

Component Knowledge

- To use understand how to round to different degrees of accuracy.
- To be able to write error intervals when rounding using correct inequality notation.
- To be able to write error intervals when rounding using correct inequality notation.

Key Vocabulary

Rounding	Rounding means making a number simpler but keeping its value close to what it was. The result is less accurate, but easier to use.
Accuracy	How close the rounded value is to the original value.
Place value	The value of the digit in a number
Lower bound	The smallest possible value that can be rounded to the number given.
Upper bound	The largest possible value the rounded value can take.
Truncation	Truncation comes from the word truncare, meaning "to shorten". The number is cut off at a certain point.
Inequality notation	Symbols used to describe the relationship between two expressions that are not equal to one another.

Inequality Notation

All error intervals look the same like this:

$$\_\_\_\_ \leq n < \_\_\_\_$$

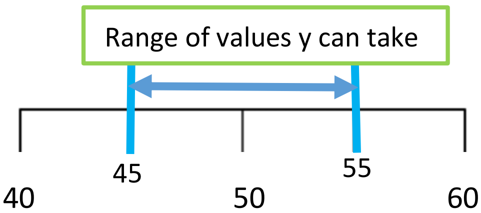
The value, n, can be greater or equal to this number.

The value, n, can only be less than this number but we use it to make any calculations easier to perform, should we need to.

Error intervals- rounding according to place value

Example 1- Frank rounds a number, y, to the nearest ten. His result is 50 Write down the error interval for y.

Begin by finding the ten, in this case, greater than and less than 50.



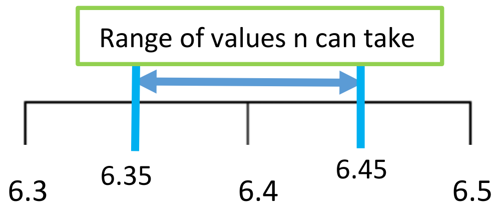
The midpoint between 40 and 50 is 45. This is the lower bound.

The midpoint between 50 and 60 is 55. This the upper bound (this can never = 55 but can be as large as 54.9999999….. 55 is easier to calculate with. Additionally, we use < as well.

The answer is $45 \leq y < 55$.

Example 2- Freya rounds a number, n, to one decimal place. Her result is 6.4 Write down the error interval for n.

Begin by finding the tenth, in this case, greater than and less than 6.4. (**Note: 1dp = tenths column.**)



The midpoint between 6.3 and 6.4 is 6.35. This is the lower bound.

The midpoint between 6.4 and 6.5 is 6.45. This the upper bound (this can never = 6.45 but can be as large as 6.49999999….. 6.45 is easier to calculate with. Additionally, we use < as well.

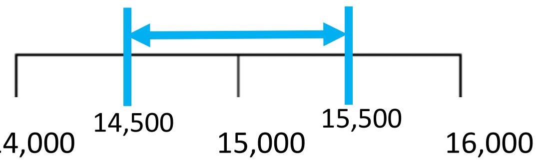
The answer is $6.35 \leq n < 6.45$.

Error intervals- rounding according to significant figures

Depending on the size of the number, the rounding will change when rounding to significant figures. Rounding like this keeps all numbers rounded to the same degree of accuracy relative to the size of the number.

Example 3- A number, g, is 15,000 when rounded to 2 significant figures. Write down the error interval.

Begin by finding the place value of the 2nd significant figure, in this case, this is 5000. This means we are rounding to 2 sig figs = rounding to nearest thousand.



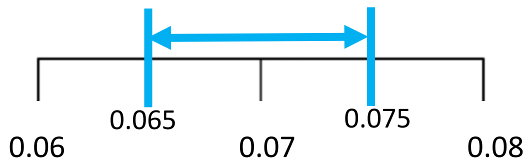
The midpoint between 14,000 and 15,000 is 14500. This is the lower bound.

The midpoint between 15,000 and 16,000 is 15,500. This the upper bound.

The answer is $14{,}500 \leq g < 15{,}500$.

Example 4- A number, x, is 0.07 when rounded to 1 significant figure. Write down the error interval.

Begin by finding the place value of the 1st significant figure, in this case, this is 0.07. This means we are rounding to 1 sig fig =rounding to nearest hundredth.



The midpoint between 0.06 and 0.07 is 0.065. This is the lower bound.

The midpoint between 0.07 and 0.08 is 0.075. This the upper bound.

The answer is $0.065 \leq x < 0.075$.

Error intervals- truncation

Be careful when reading error interval questions as truncating is not rounding like place value. The number has been "chopped", which means the value given **IS THE LOWER BOUND.** It commonly applies to decimals.

Example 5- State the error interval of 4.5 when it has been truncated to 1 decimal place.

Begin by finding the tenth, in this case, greater than 4.5. (**Note: 1dp = tenths column.)** This is the upper bound.

Remember: the value cannot equal 4.6!

The answer is $4.5 \leq n < 4.6$.

Online clip

M730

Estimation

Component Knowledge

- Estimate values of numeric problems
- Estimate values of worded problem solving questions
- Identify whether an estimation is an under-estimate or an over-estimate

Key Vocabulary

Round	Making a number simpler whilst keeping its value close to the original.
Significant figures	The number of digits in a value that carry a meaning to the size of the number.
Estimate	Find a value that is close to the right answer by rounding.

When estimating any calculation, you need to round every number to one significant figure

Estimating Calculations

Estimate 39 x 4.85

$$39 \times 4.85$$

$$40 \times 5$$

$$= 200$$

Estimate

$$\frac{52 \times 6.78}{0.51}$$

First round all the numbers to 1 significant figure

$$\frac{50 \times 7}{0.5}$$

Then calculate the numerator

$$\frac{350}{0.5}$$

Dividing by 0.5 is the same as multiplying by 2

$$750$$

Significant figures

Example

Round 3786 to one significant figure

Th	H	T	U
3	7	8	6

The first significant figure is in the thousands column so to the nearest thousand it is 4000

Estimation worded problems

Mr Sykes wants to buy a calculator for every student in year 11. There are 105 students in year 11. Each calculator costs £6.99

Work out an estimate for the amount of money Mr Sykes will spend on calculators.

First round all the numbers to 1 significant figure

$105\ students$ → $100\ students$

$£6.15$ → $£6$

$$100 \times £6 = £600$$

Online clips

M994, M131,M878

How to decide if your solution is an underestimate or overestimate.

Decide if you have made each number bigger or smaller by rounding. When dividing remember that if you divide by a number that has been rounded up, your answer will be an underestimate and vice versa

For example: In the calculator example above we rounded the cost and number of students down so this is an under estimate of the cost.

Fractions, decimals, & Percentages

Component Knowledge

- Convert between simple fractions, decimals and percentages.
- Order fractions, decimals and percentages by converting.

Key Vocabulary

Fraction	Made up of a numerator (top) and denominator (bottom). Compares parts in question to total number of parts.
Integer	Whole number
Ascending order	Place numbers in order from smallest to largest
Descending order	Place numbers in order from largest to smallest
Percentage (percent)	'Out of' (per) one hundred (cent)
Decimal	Comparable number to a fraction or mixed number, written using place value, e.g. $\frac{2}{5} = 0.4$, or $3\frac{3}{4} = 3.75$

Convert % to fraction:

$$\% \text{ "means out of 100"} = \frac{\quad}{100}$$

$$eg\ 65\% = \frac{65}{100}\ simplify\ where\ possible$$

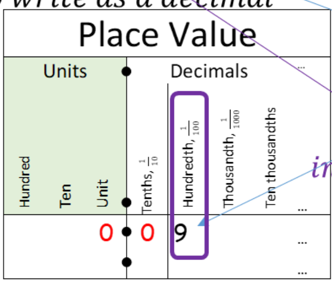
$$= \frac{65}{100} = \frac{13}{20} \quad (\div 5)$$

Convert % to fraction to decimal:

$$Convert\ to\ fraction\ out\ of\ 100, \frac{\quad}{100}$$

$$as\ \% \text{ "means out of 100"} = \frac{\quad}{100}$$

$$eg\ 9\% = \frac{9}{100}$$ use place value table to write as a decimal



place the 9

in the hundredths column

fill in with any zeros

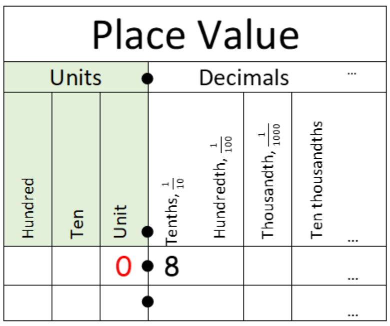
Convert decimal to a fraction

Use place value to convert to fraction out of 10, 100, 1000, etc

$$eg\ 0.8 = \frac{8}{10}$$

then simplify where possible

$$eg\ \frac{8}{10}\ becomes\ \frac{4}{5}$$



Convert decimal to a fraction to a percentage

Use place value to convert to fraction out of 10, 100, 1000, etc

$$eg\ 0.126 = \frac{126}{1000}$$

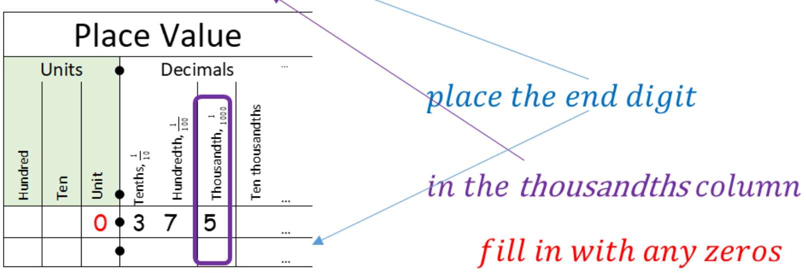
% means out of 100 so convert to equivalent fraction out of 100 $= \frac{\quad}{100}$

$$eg\ \frac{126}{1000}\ becomes\ \frac{12.6}{100} = 12.6\%$$

Convert fraction to decimal

$Convert\ to\ fraction\ out\ of\ 10, 100, 1000, etc" = \frac{}{10}\ or\ \frac{}{100}\ or\ \frac{}{1000}$
$then\ use\ place\ value\ to\ write\ as\ a\ fraction$

$eg\ \frac{3}{8} = \frac{\times}{10} = \frac{\times}{100} = \frac{375}{1000}$



Convert fraction to percentage

$Convert\ to\ fraction\ out\ of\ 10, 100, 1000, etc" =$

$\frac{}{10}\ or\ \frac{}{100}\ or\ \frac{}{1000}$

$eg\ \frac{3}{200} = \frac{\times}{10} = \frac{\times}{100} = \frac{15}{1000}$

$then\ write\ as\ an\ equivalent\ fraction\ "out\ of\ 100"\ as\ percentage$

$eg\ \frac{15}{1000} = \frac{1.5}{100}$ (÷ 10 top and bottom) once "out of 100" write as a percentage = 1.5%

Ordering FDP

To be able to order FDP, we need to write them all in the same format.

Example: Order from smallest to largest $\frac{1}{4}$ 0.19 0.3 26% $\frac{1}{5}$

You can choose to convert them all into fractions, decimals or percentages as long as you convert them all into the same.

Changing them to percentages:

$\frac{1}{4}$ = 25% 0.19 = 19% 0.3 = 30% $\frac{1}{5}$ = 20%

25%, 19%, 30%, 26%, 20%

Rewrite the list with the numbers all in the same format.

From smallest to biggest:

19%, 20%, 25%, 26%, 30%

Answer:

0.19, $\frac{1}{5}$, $\frac{1}{4}$, 26%, 0.3

Make sure you write your answer using the original numbers in the question.

Online clips

M958, M264, M553

Express a Quantity as a Fraction of Another

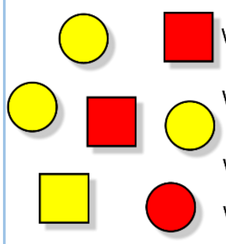
Component Knowledge

- Express one quantity as a fraction of another
- Express a fraction in its simplest form

Key Vocabulary

Fraction	A way to express a part of a whole
Amount	The sum total of 2 or more quantities of sums
Quantity	A certain amount or number of something
Numerator	To top number of a fraction showing how many parts of the whole
Denominator	The bottom number that names the fraction
Simplify	To reduce a fraction to a simpler form by diving the numerator and denominator by a common factor
Proportion	The comparison of the size of a share to the size of the whole

Simple Examples



What fraction of these shapes are **red**? $\frac{3}{7}$

What fraction of these shapes are **circles**? $\frac{4}{7}$

What fraction of the **circles** are **red**? $\frac{1}{4}$

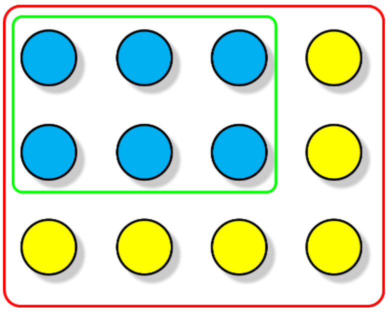
What fraction of the **squares** are **yellow**? $\frac{1}{3}$

What fraction of the **red shapes** are **squares**? $\frac{2}{3}$

What fraction of the **yellow shapes** are **circles**? $\frac{3}{4}$

Expressing a Quantity as a Fraction in its simplest form

What fraction of these shapes are **blue**?



parts → whole →

$$\frac{6}{12} = \frac{1}{2}$$

Applied Examples

Marc buys a bag of 30 sweets.

6 of the sweets are apple... Marc hates apple!

What fraction of the sweets will Marc not eat?

simplify (÷6)

parts → whole →

$$\frac{6}{30} = \frac{1}{5}$$

one-fifth of the sweets

1 out of every 5 sweets

Eric ran 500 metres.

Dani ran 1.6 kilometres

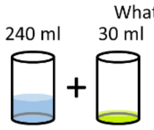
How far did Eric run as a fraction of how far Dani ran?

Eric → Dani →

$$\frac{500}{1600} = \frac{5}{16}$$

We must use the same units in the fraction.

What fraction of the drink is juice?



$$\frac{30}{240 + 30} = \frac{30}{270} = \frac{1}{9}$$

Online clips

U163

Percentages

What do I need to be able to do?

- Be able to write a quantity as a percentage of another
- Be able to find percentages of an amount.
- To be able to find a percentage increase and decrease
- Be able to find a percentage change.
- To be able to use reverse percentages.

Key Vocabulary

Percentage	Parts per 100. The unit is %
Equivalent	Having the same value
Convert	The change a value or expression from one form to another
Fraction	How many parts of a whole
Decimal	Based on 10. Decimal number is often used to mean a number that uses a decimal point
Growth	To increase/to grow
Reduce	To make smaller in value

To calculate any percentage it is useful to start with 10%.

30% of 120: 10% = 120 ÷ 10 = 12 — To find 10% we divide by 10.

30% = 3 x 12 = 36 — To find 30% we multiply 10% by 3.

80% of 120: 80% = 0.80

80% of 120 = 0.80 x 120 = 96 — Change the percentage to a decimal and then multiply.

33% of 90: 33% = 0.33

33% of 90 = 0.33 x 90 = 29.7

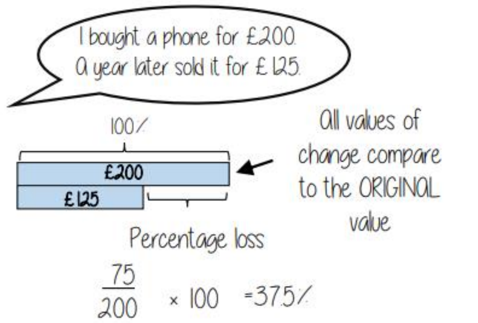
4% of 88: 4% = 0.04 — Be careful if the percentage is less than 10.

4% of 88 = 0.04 x 88 = 3.52

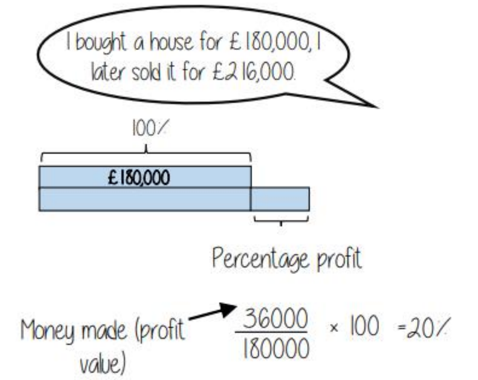
12.5% of 42: 12.5% = 0.125 — Take care using decimal percentages, still divide by 100.

12.5% of 42 = 0.125 x 42 = 5.25

Percentage change



$$\frac{\text{Difference in values}}{\text{Original value}} \times 100$$



Percentage Increase and Decrease

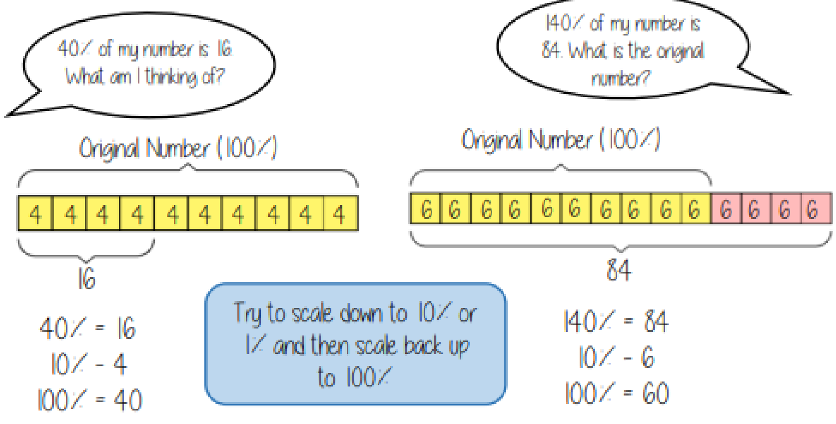
Example 1) Increase £320 by 20%.

Work out 20%, (0.20 x 320 = 64) Add this onto of the original number, 320 + 64 = £384

Example 2) Decrease £50 by 12%.

12% = 0.12 x 50 = 6. Subtract this from 50 = 50 - 6 = £44.

Reverse Percentages



Online Clips: **M235, M437, M905, M476, M533, M528**

Expanding single brackets

Component Knowledge

To be able to expand a single bracket, including problems with powers.

Key Vocabulary

Expression	A mathematical statement written using symbols, numbers or letters.
Simplify	In general, an expression is in simplest form when it is easiest to use
Expand	Expand is when we multiply to remove the ( )

Expanding brackets means multiplying everything inside the bracket by the letter or number outside the bracket.
For example, in the expression $3(m+7)$ both m and 7 must be multiplied by 3:

$3(m+7)$
$=3 \times m + 3 \times 7$
$=3m + 21$

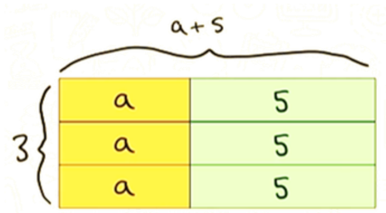
Expanding brackets involves using the skills of simplifying algebra. Remember that $2 \times a = 2a$

Example

Expand $4(3n+y)$.
$=4 \times 3n + 4 \times y$
$= 12n + 4y$

Using grid method

Expand: 3(a + 5)   3 x (a + 5)



3 x a = 3a   3 x 5 = 15

3a + 15

Using arrows

Expand:

7 (3 + a) = 21 + 7a

$3x(5x+2) = 15x^2 + 6x$

Expanding and simplifying

To expand and simplify more than one bracket, first expand each bracket then collect like terms.

2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a
= 5a + 16

Note – collect like terms to simplify

$4(x+2) - 2(x+2) = 4x + 8 - 2x - 4$
$= 2x + 4$

Note: Remember the rules when multiplying negatives, - 2 multiplied by x = -2x

Online clips

M237, M792

Expanding Double Brackets

Component Knowledge

- To use algebraic notation when multiplying terms.
- To be able to expand double brackets and simplify where necessary.
- Use identity notation correctly.

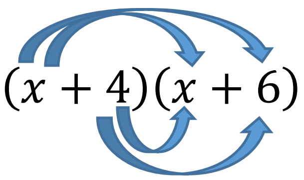
Key Vocabulary

Expand	Multiplying out a bracket.
Term	Either a single number or variable, or the product of several numbers or variables.
Collecting like terms	Simplifying an expression by grouping the same type of terms together.
Identity	An equality that relates one variable to another. It will be equal for ALL values of the variable, unlike an equation which gives a single solution.

Expanding double brackets

Expanding double brackets is long multiplication using algebraic terms as well as numerical values. There are 2 common ways of completing this.

Example 1 -Expand $(x+4)(x+6)$



We multiply all terms together (this can be known as FOIL method:

$$x \times x = +x^2$$

$$x \times 6 = +6x$$

$$4 \times x = +4x$$

$$4 \times 6 = +24$$

$$(x+4)(x+6) \equiv x^2 + 6x + 4x + 24$$

We now collect like terms:

$$\equiv x^2 + 10x + 24$$

Example 1 -Expand $(x+4)(x+6)$

We can also use an area model (also known as the grid method).

X	x	+4
x	$+x^2$	+4x
+6	+6x	+24

We have still multiplied all the terms together, like the previous method, but they remain in the grid. We can see all 4 terms in the expanded expression:

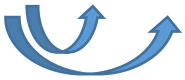
$(+x^2 + 6x + 4x + 24)$.

We now collect like terms:

$$(x+4)(x+6) \equiv +x^2 + 10x + 24$$

Example 2 -Expand $(x-3)(x+6)$

$$(x-3)(x+6)$$



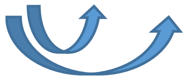
This gives $x^2+6x-3x-18$

This simplifies to

$x^2+3x-18$

Example 3 -Expand $(2x-3)(x-5)$

$$(2x-3)(x-5)$$



This gives $2x^2-10x-3x+\ 15$

This simplifies to

$2x^2-13x+15$

Online clips

Q976, M527

Function machines and solving 1 and 2 step equations

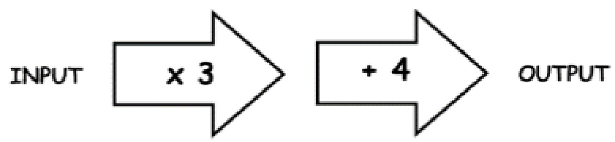
Component Knowledge

- To be able to use function machines to find the input and output value.
- To be able to solve one-step equations.
- To be able to solve two-step equations.

Key Vocabulary

Function Machine	Takes an input value, performs some operations and produces an output value.
Operation	Common operations are addition, subtraction, multiplication and division.
Inverse	The operation of another function.
Equation	a mathematical statement that shows that two mathematical expressions are equal
Solve	To find the solution

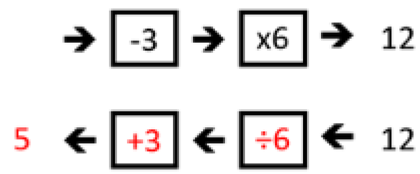
Function Machines



If the input is 5 the calculation is

5 x 3 = 15

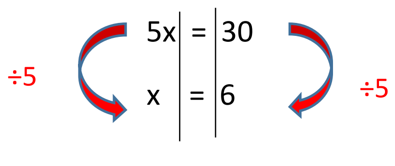
15 + 4 = 19

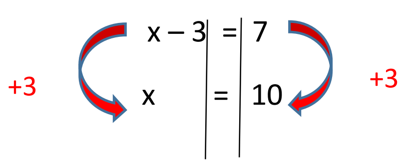


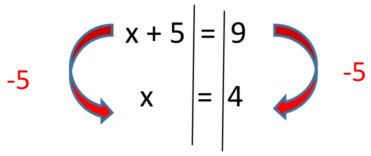
To find the input, start with the input and work backwards doing the inverse operations of the function machine.

One- step equations

To solve a one-step equation, you need to do the inverse operation.



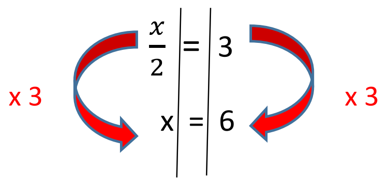




The inverse of multiplying is dividing. We divide 30 by 5.

The inverse of subtracting is addition. We add 3 to 7.

The inverse of addition is subtraction. We subtract 4 from 9.



The inverse of dividing is multiplying.

We multiply 2 by 3.

Two- step equations

To solve a one-step equation, you need to do the inverse operation.

To solve a two-step equation or inequality we need to complete 2 inverse calculations in a specific order.

$6x + 3 = 32$

-3 The inverse of adding 3 is subtracting 3

$6x = 30$

÷ 6 The inverse of multiplying 6 is dividing by 6

$x = 5$

$4x - 3 = 13$

+3 The inverse of subtracting 3 is adding 3

$4x = 16$

÷ 4 The inverse of multiplying 4 is dividing by 7

$x = 4$

$\frac{x-5}{3} = 4$

÷ 3 The inverse of dividing by 3 is multiplying by 3

$x - 5 = 12$

+ 5 The inverse of subtracting 5 is adding 5

$x = 17$

Online clips

M175, M428, M707, M634, M647, M855, M401

Solve equations unknown on both sides

Component Knowledge

- Solve equations with unknown on both sides
- Solve equations with unknown on both sides with brackets
- Solve equations with unknown on both sides with fractions
- Solve equations with unknown on both sides after forming

Key Vocabulary

Solve	To find a value (or values) that satisfies an equation
Equation	An equation says that two things are equal
Brackets	Symbols used in pairs to group things together
Inverse	The operation that reverses the effect of another operation
Constant	The number on its own, not attached to a letter

Solve equations with unknown on both sides

Solve $5x + 1 = 2x - 8$

$-2x \qquad -2x$

Step 1: Collect the variables (like terms) together, to do this find the inverse operation of the smallest number of x's. Whatever you do to one side, do the same to the other side.

$3x + 1 = -8$

$-1 \qquad -1$

Step 2: Complete the inverse operations to find x. Always follow the inverse of the order of operations (Bidmas). Here subtract 1 is the inverse of adding 1.

$3x = -9$

$\div 3 \qquad \div 3$

Step 3: Continue to find the inverse operations. Divide by the number in front of the unknown.

$x = -3$

Solve equations with unknown on both sides involving brackets

$4(2x + 5) = 2(6x - 2)$

$8x + 20 = 12x - 4$

Expand brackets first

$-8x \qquad -8x$

Collect the variables by finding the inverse operation of the smallest number of x

$20 = 4x - 4$

$+4 \qquad +4$

Continue to use inverse operations to find x. Here add 4 to each side as the inverse of subtract 4.

$24 = 4x$

$\div 4 \qquad \div 4$

Continue to use inverse operations to find x. Here divide by 4 as the inverse to multiply by 4.

$6 = x$

Solve equations with unknown on both sides with fractions

$$3x - 15 = \frac{x + 11}{3}$$

$\times 3 \qquad \times 3$

$$3(3x - 15) = x + 11$$

$$9x - 45 = x + 11$$

$-x \qquad -x$

$$8x - 45 = 11$$

$+45 \qquad +45$

$$8x = 56$$

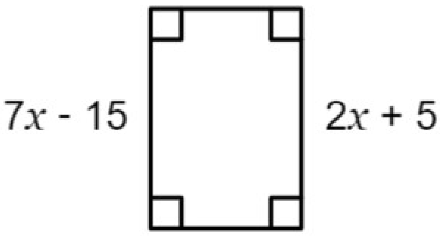
$\div 8 \qquad \div 8$

$$x = 7$$

Start by removing any values on the denominator. To do this complete the inverse operation. Here the inverse of divide by 3 is to multiply both sides by 3. Remember every term on the LHS must be multiplied by 3. You can use expanding brackets here to remember to multiply both terms by 3.

Solve equations with unknown on both sides after forming it

Find the value of x.



This is a rectangle. Both of the sides with expressions on them are equal to each other so you can form an equation to show this and then solve as before.

$$7x - 15 = 2x + 5$$

$-2x \qquad -2x$

$$5x - 15 = 5$$

$-15 \qquad -15$

$$5x = 20$$

$\div 5 \qquad \div 5$

$$x = 4$$

Online clips

M554, M957

Inequalities

Component Knowledge

- Understand and use inequality notation
- Represent the solution set of an inequality on a number line
- Decide whether a number satisfies an inequality
- Form an inequality from a question and solve it

Key Vocabulary

Inequality	An inequality shows that two quantities are (may) not be equal
Less than	This is shown by the symbol $<$
Less than or equal to	This is shown by the symbol $\leq$
Greater than	This is shown by the symbol $>$
Greater than or equal to	This is shown by the symbol $\geq$
Integer	A whole number

Notation

$x > 2$ means x **is greater than 2**

$x < 3$ means x **is less than 3**

$x \geq 1$ means x **is greater than or equal to 1**

$x \leq 6$ means x **is less than or equal to 6**

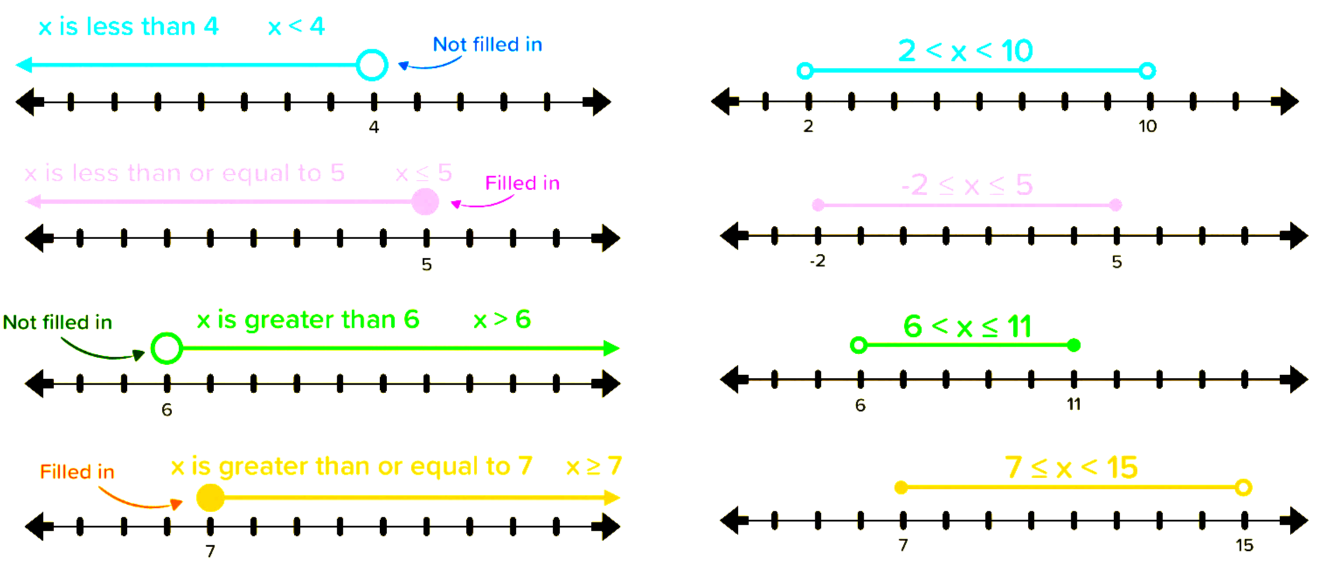
Examples:

$x \geq 1$ is **true** for $x = 6, 2.5$ and 1

$x < 5$ is **false** for $x = 10, 5.05$ and 5

The set of *integers* which **satisfy** $-2 \leq x < 3$ is $\{-2, -1, 0, 1, 2\}$

The set of numbers *satisfying* an inequality can be *represented* on a number line:



Inequalities can be **solved** by the same method as used for equations:

a) $x - 7 \leq 12$

b) $5y > 40$

c) $\frac{b}{3} \geq -2$

One-step solution

$$x - 7 \leq 12 \quad (+7)$$
$$x \leq 19$$

$$5y > 40 \quad (\div 5)$$
$$y > 8$$

$$\frac{b}{3} \geq -2 \quad (\times 3)$$
$$b \geq -6$$

Inverse operation

a) $5(x - 1) < 3.5$

$\frac{b}{6} + 2 \geq 1$

Two-step solution

$$5(x - 1) < 3.5 \quad (\div 5)$$
$$x - 1 < 0.7 \quad (+1)$$
$$x < 1.7$$

$$\frac{b}{6} + 2 \geq 1 \quad (-2)$$
$$\frac{b}{6} \geq -1$$
$$b \geq -6$$

Make sure you write an inequality symbol

<u>Online clips</u>

M384, M118

Rearranging simple formulae

Component Knowledge

- To rearrange formulae using one inverse operation.
- To rearrange formulae using two inverse operations.

Key Vocabulary

Formula	expresses the relationship between two or more unknown values
Term	either a single number or variable, or the product of several numbers or variables
Rearrange	change the form of the equation to display it in a different way

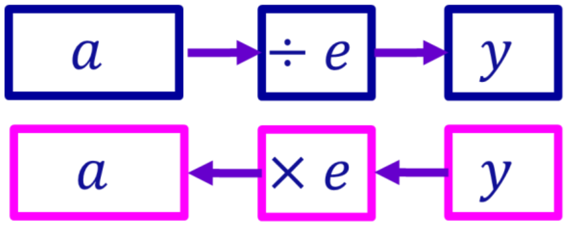
Rearranging one step equations

Make a the subject of the formula:

$$\frac{a}{e} = y$$

giving $ye = a$

We want to make a the subject, so read from the letter a.
We can draw a function machine to help us rearrange.



We can also use "tramlines" to rearrange the equation.

$$\frac{a}{e} = y$$

$\times e \qquad \times e$

$a = ye$

Further Examples:

Make x the subject (change the order of the terms so 'x' is on its own)

$x + y = c$

$x = c - y$ (−y)

The inverse of adding y to is subtracting y.

Subtract y from both sides to result in x being made the subject.

$3x = b$

$x = \frac{b}{3}$ (÷ 3)

The inverse of multiplying 3 by x is dividing by 3.

Divide both sides by 3 to result in x being made the subject.

Rearranging Two step equations:

Make a the subject of the formula:

$$\frac{a+b}{e} = y$$

giving $ye - b = a$

We want to make a the subject, so read from the letter a.
We can draw a function machine to help us rearrange.

a → $+b$ → $\div e$ → y

a ← $-b$ ← $\times e$ ← y

We can also use "tramlines" to rearrange the equation.

$$\frac{a+b}{e} \;\Big|=\Big|\; y$$

$$\times e \qquad \times e$$

$$a+b \;|=|\; ye$$

$$-b \qquad -b$$

$$a \;|=|\; ye - b$$

Further Examples

Make x the subject (change the order of the terms so 'x' is on its own)

$\frac{x}{6} + 5 \;|=|\; w$ -5

$\frac{x}{6} \;|=|\; w - 5$ x 6

$x \;|=|\; 6(w - 5)$

The first step is to do the inverse of adding 5 which is subtracting 5.

The second step is to do the inverse of dividing by 6 which is multiply by 6.

We use brackets because both the w and the -5 need to be multiplied by 6.

$x^2 - 3 \;|=|\; h$ +3

$x^2 \;|=|\; h + 3$ $\sqrt{\;}$

$x \;|=|\; \sqrt{h+3}$

The first step is to do the inverse of subtracting 3 which is adding 3.

The second step is to do the inverse of squaring a number which square root

Online clips

M242, M983

Substitution

Component Knowledge

- To substitute positive and negative numbers into expressions with one, or more, variables.

Key Vocabulary

Expression	A maths sentence that includes a minimum of 2 variables, including an algebraic term and at least one operation.
Term	Either a single number or variable, or the product of several numbers or variables.
Substitute	To exchange an unknown variable for a number in an expression/equation/formula.

Substitution-formula

For example: The time in minutes to cook a chicken is given by the formula:

Time = 40 minutes per kilogram plus 20 minutes

Find how long it takes to cook a 5kg chicken.

Here we substitute 5kg into the formula. **So, Time= 40 x 5 +20 = 220 minutes**

The formula for speed is shown: $$Speed = \frac{Distance}{Time}$$

Find the average speed when travelling 150 miles in 4 hours.

Here we substitute Distance = 150 and Time = 4 into the formula. $Speed = \frac{150}{4} = 37.5mph$

Substitution-expressions

Example 1

$f = p + 4$. find the value of f when $p = 6$.

We substitute 6 for p in the formula.

$f = (6) + 4$

$f = 10$

Example 2

$f = 2p + 4$. find the value of f when $p = -6$.

We substitute -6 for p in the formula.

$f = 2(-6) + 4$

$f = -8$

Example 3

$f = t^2$. find the value of f when t = -6.

We substitute -6 for t in the formula.

$f = (-6)^2$

$f = 36$

Example 4

$f = \frac{t^2}{5y}$. find the value of f when t = -6, y = 4.2

We substitute -6 for t and 4.2 for y in the formula.

$f = \frac{(-6)^2}{5(2.4)}$

$f = \frac{36}{12}$

When substitute negative numbers, we must put brackets around them to ensure the correct order of operations occurs. **This very important when we use calculators.** (We can also do this with positive numbers)

From example 4. $-6^2 = -(6)^2 = -36$ is not equal to $(-6)^2 = -6 \times -6 = 36$.

Online clips: M417, M327, M208, M979

Area of trapezia & Compound shapes

Component Knowledge

- Recall area of basic 2D shapes
- Find the area of a trapezium
- Find the area of compound shapes (excluding parts of circles)

Key Vocabulary

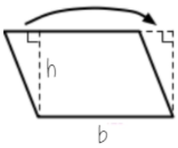
Area	The amount of space inside the boundary of a 2D shape
Congruent	When two shapes are identical, except one may be in a different orientation (way round)
Formula	A rule or fact written in mathematical symbols (algebra)
Perpendicular	At right angles (90^0) to
Compound	Made up of more than one thing (i.e. two or more shapes 'fitted' together)
Units squared (or squared units)	Unit of area, example mm^2, cm^2, m^2

Area of rectangles, triangles & parallelograms

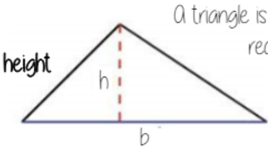
Area – rectangles, triangles, parallelograms R

Rectangle
Base x Height

Parallelogram/ Rhombus
Base x Perpendicular height



Triangle
½ x Base x Perpendicular height

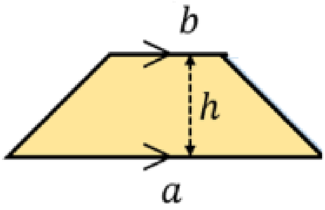


A triangle is half the size of the rectangle it would fit in

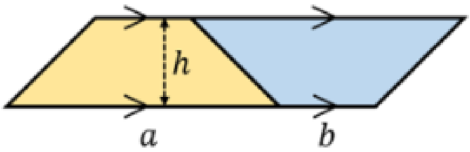
Area of a trapezium

Area of a trapezium

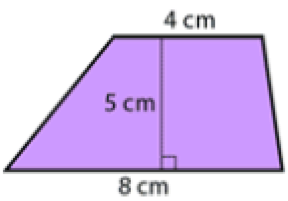
$$\frac{(a+b)\times h}{2}$$



Why?



- Two congruent trapeziums make a parallelogram
- New length (a + b) x height
- Divide by 2 to find area of one



Area= 4 + 8 = 12 ← Add the parallel sides.

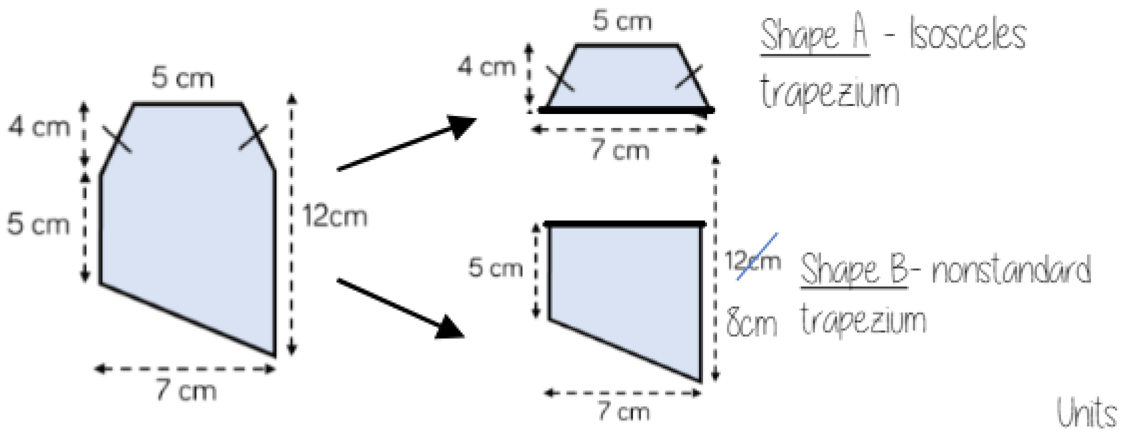
12 ÷ 2 = 6 ← Divide the total by 2.

6 x 5 = 30cm² ← Multiply by the height.

Compound shape example

Compound shapes

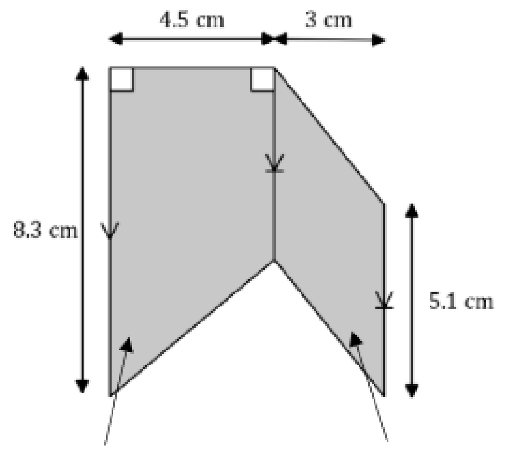
To find the area compound shapes often need splitting into more manageable shapes first. Identify the shapes and missing sides etc. first.



Shape A + Shape B = total area

$\frac{(5+7) \times 4}{2} + \frac{(5+8) \times 7}{2} = 24 + 45.5 = 69.5cm^2$

Further example



Area of the trapezium is:

$\frac{1}{2} \times (8.3 + 5.1) \times 4.5 = 30.15cm^2$

Area of the parallelogram is:

$5.1 \times 3 = 15.3cm^2$

So the total area of the composite shape is:

$30.15 + 15.3 = 45.45\ cm^2$

Online clips

M705, M996, M303

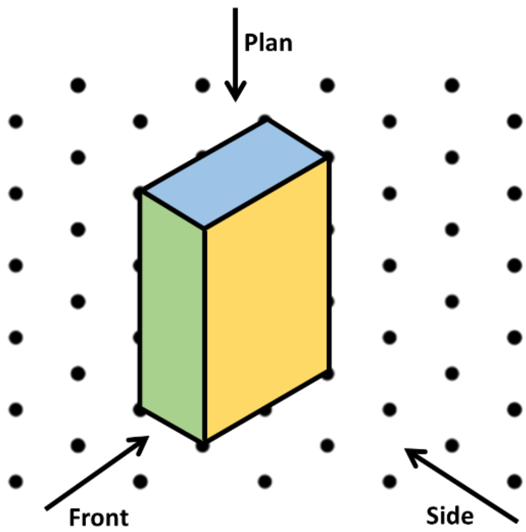
Plans and elevations

Component Knowledge

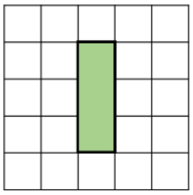
- Draw the plan of an oriented 3-dimensional shape
- Draw the front elevation (direction specified) of a 3-dimensional shape
- Draw the side elevation (direction specified) of a 3-dimensional shape

Key Vocabulary

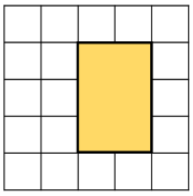
Plan	The view of an oriented 3-dimensional shape from above
Front elevation	The view of an oriented 3-dimensional shape from a specified front direction
Side elevation	The view of an oriented 3-dimensional shape from the side



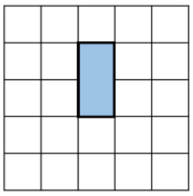
Front Elevation



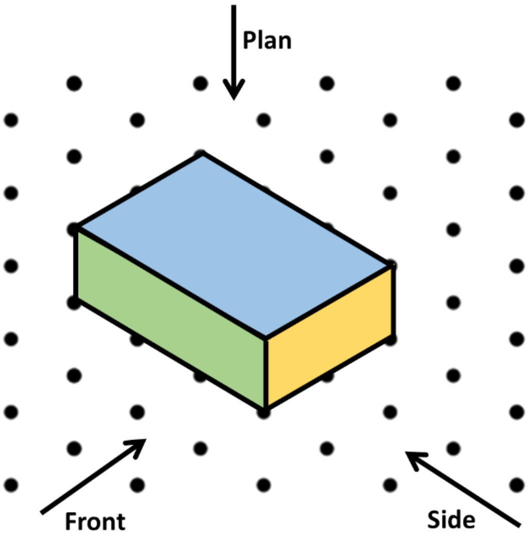
Side Elevation



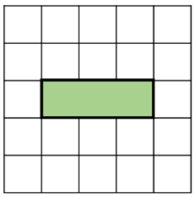
Plan



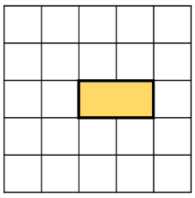
Ensure that the dimensions of the plan and the elevation are consistent with the lengths of the 3-dimensional shape.



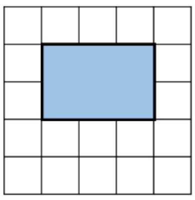
Front Elevation



Side Elevation



Plan

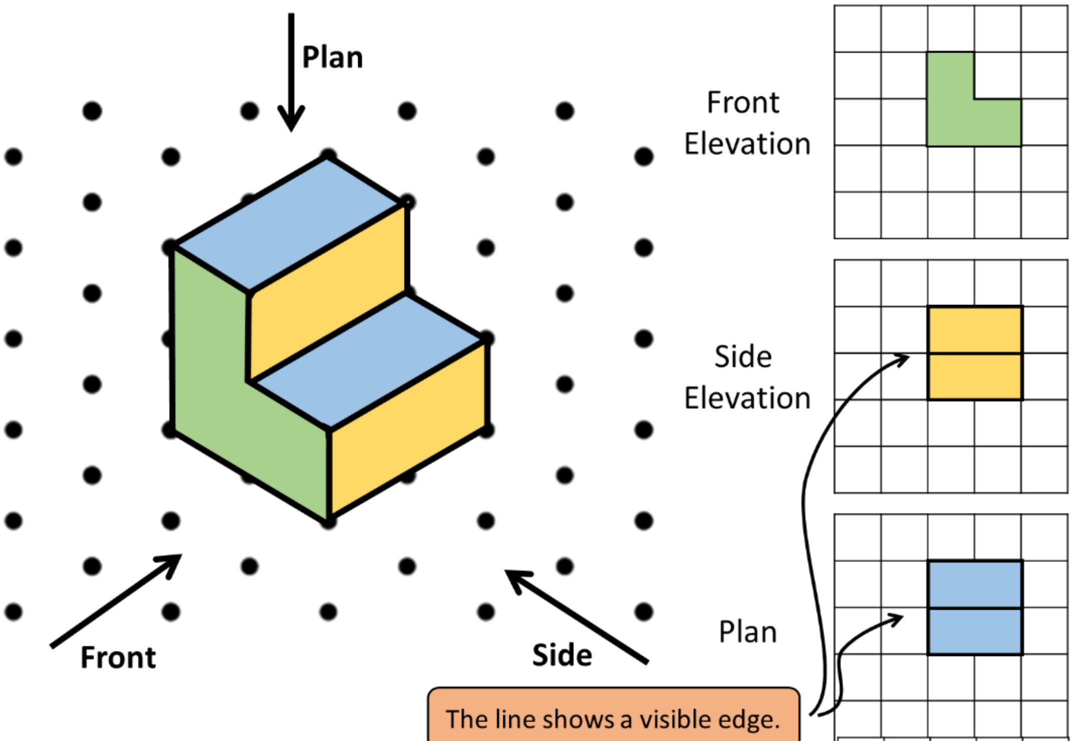


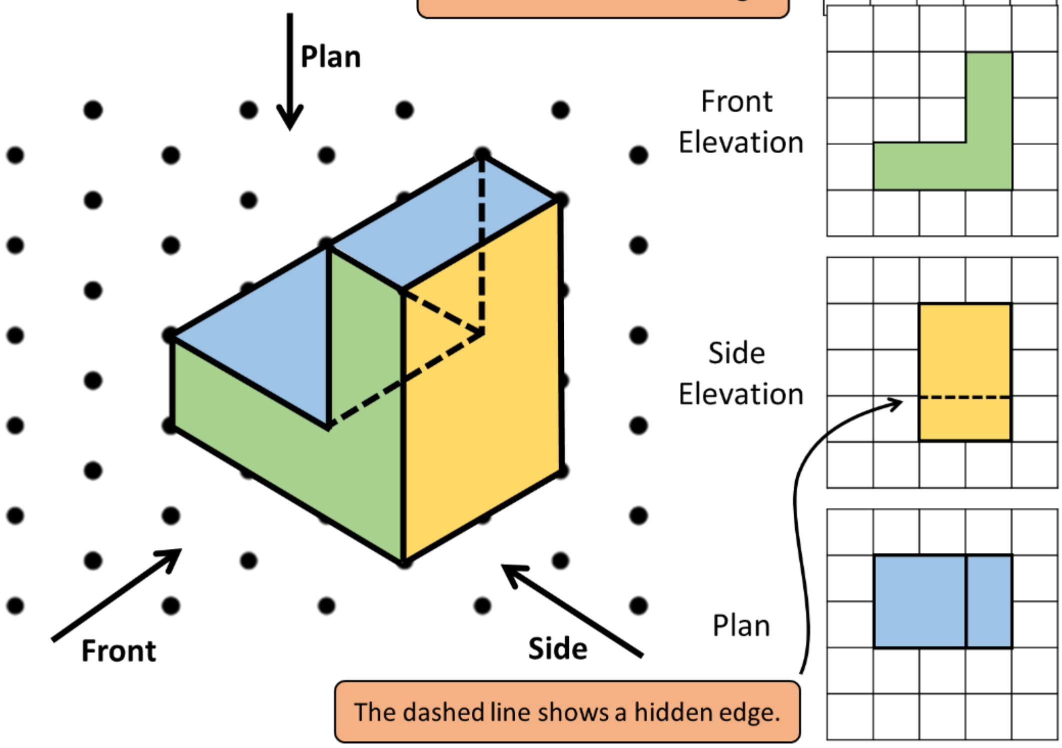
If only the front direction is specified, both the left and right-side view are acceptable as the side elevation.

(They are either the same, or mirror images)

Showing edges in plans and elevations

This provides more information about the shapes and makes it easier to identify the direction from which the plan and elevation are drawn.





Online clips

M229

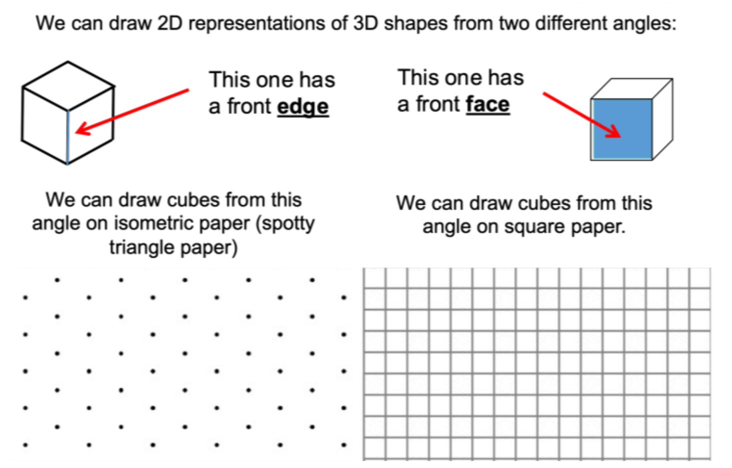
Isometric Drawing

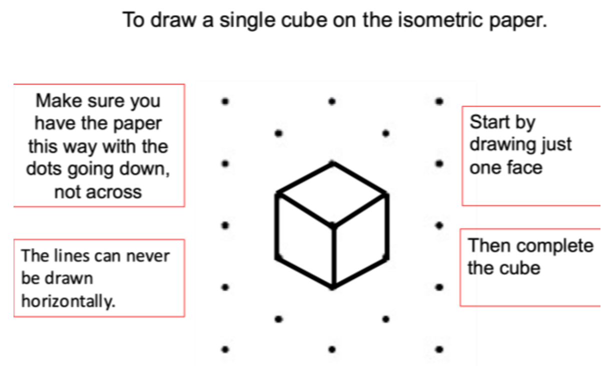
Component Knowledge

- To be able to draw a 3D shape on Isometric paper

Key Vocabulary

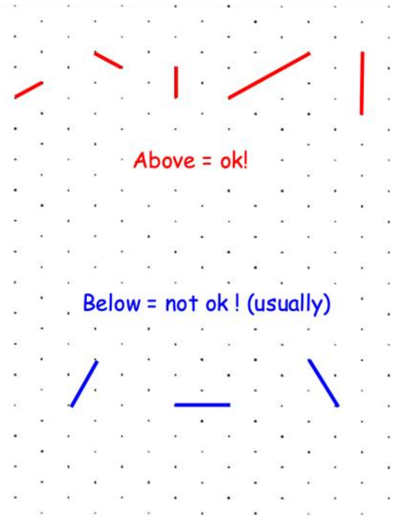
Isometric	An isometric drawing is a drawing of a 3- dimensional shape on a two-dimensional surface. A vertical line is used as a place to start. Horizontal lines are created at 30- degree angles.
Isometric Paper	Isometric paper is paper with dots arranged in equilateral triangles.
Edge	An edge is where two faces, on a shape, come together. On 3D shapes they are the lines that separate each face.
Vertex	A vertex is a corner where edges meet.
Faces	A face is a flat or curved surface on a 3D shape.

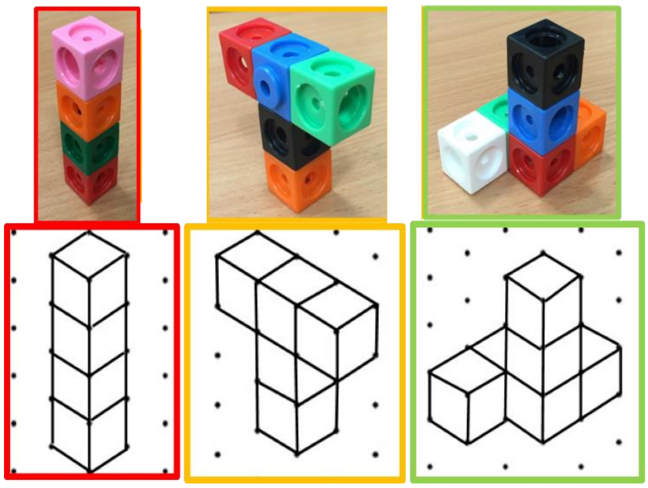




When drawing objects on isometric paper, you very rarely (if ever) join dots across wider gaps

They usually join to dots directly next to them…





Online clips