Sequences

Component Knowledge

- Understand and describe linear sequences.
- Understand and describe geometric sequences.
- Understand and describe pictorial sequences.
- Understand and describe special sequences

Key Vocabulary

Linear	The difference between terms increases or decreases by the same value each time.
Geometric	A sequence where each term is found by multiplying the previous one by a fixed non-zero number.
Difference	The gap between two terms.
Sequence	Items or numbers put in a pre-decided order
Term	A single number or variable.

Linear Sequences- can also be known as arithmetic sequences. Where each term is added or subtracted by the same amount each time (arithmetic operation).
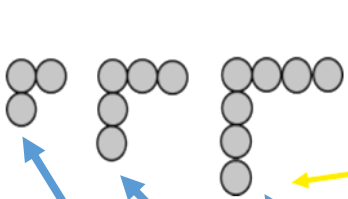
The rule is add 4 because the difference between each number is 4.

1, 5, 9, 13, ... 17

+4 +4 +4

17 is the next number because 13 + 4 = 17.

Sequences

Finding the next term - diagrams

The 4th pattern would look like this.

We can see an extra dot has been added along the top and down the side each time.

3, 5, 7, 9 …

Often patterns of shapes can be simplified to a number sequence.

Geometric Sequences- where each term is multiplied or divided by the same value each time.

Example:

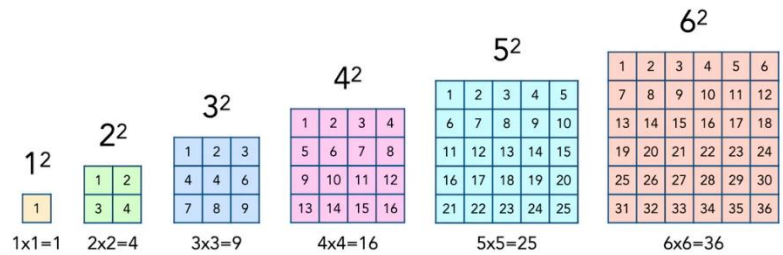The rule is multiply by 2 because the numbers are doubling.

5, 10, 20, 40, ... 80

x2   x2   x2

80 is the next number because 40 x 2 = 80

Special Sequences

Square Number Sequences-each term is the result of each position multiplied by itself. They can be represented by a square.

Example: 1, 4, 9, 16, 25, 36 …

1^2 2^2 3^2 4^2 5^2 6^2

1x1=1   2x2=4   3x3=9   4x4=16   5x5=25   6x6=36

Triangular Number Sequences-Triangular numbers are numbers that can be represented as a triangle. To find the next triangular number in a sequence, we need to find the difference between the previous two terms and add one more than this value.

Example:

+2   +3   +4   +5   +6   +7

1,   3,   6,   10,   15,   21,

1	1 + 2	1 + 2 + 3	1 + 2 + 3 + 4
1	= 3	= 6	= 10
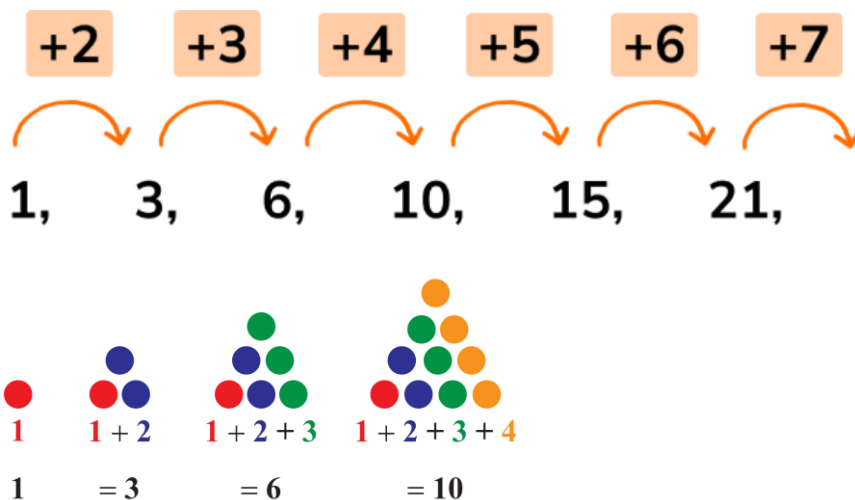
Fibonacci Sequences- This sequence is commonly found in spirals in nature. The next term is found by adding up the two terms before it.

Example: The Fibonacci sequence is below. Other Fibonacci type sequences are based on the Fibonacci rule of adding the previous 2 terms.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …

0, (0+1), (1+1),(1+2), (2+3), (3+5), …

Online clips   M381, M241, M166, M981
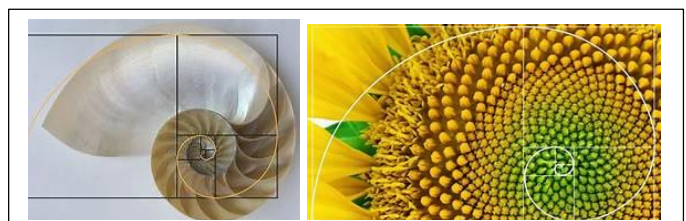
Finding and using the nth term

Component Knowledge

- Find the common difference between terms in a sequence
- Using the common difference to find the nth term
- Using the nth term to find terms in a sequence

Key Vocabulary

Sequence	A list of numbers or objects in a special order
Linear	A sequence where each term is added, or subtracted, by the same amount each time.
Pattern	Objects or numbers that are arranged following a rule or rules
Nth Term	A formula that enables us to find any term in a sequence
Term	In algebra, a term is either a single number or variable, or numbers and variables multiplied together.

How to find common differences & the nth term of a linear sequence

The nth term is the general rule for a sequence. We can use the nth term to then calculate any term in the sequence.

Here is a sequence: 5, 8, 11, 14, …

1. Find the difference between the numbers.

2. Calculate how you get from the times table to the original sequence.

A difference of +3 means we need to look at the +3 times table.

$$5, 8, 11, 14$$

+3 +3 +3 = 3n

3, 6, 9, 12, …

5, 8, 11, 14, …   + 2

We can also write this as

$$t_n = 3n + 2$$

The nth term is 3n + 2.

Decreasing sequences – follow the same steps but your nth term will be negative

A difference of -3 means we need to look at the -3 times table.

-3 -3 -3 -3

-5, -8, -11, -14, -17

-2   -2

-3n: -3 -6 -9 -12 -15

$$-3n - 2$$

Calculate how you get from the times table to the original sequence

Using the nth term to create a sequence

Write the first five terms of the sequence $3n + 4$.

n represents the position in the sequence. The first term in the sequence is when $n = 1$, the second term in the sequence is when $n = 2$, and so on.

To find the terms, **substitute** n for the position number:

- when $n = 1, 3n + 4 = 3 \times 1 + 4 = 3 + 4 = 7$
- when $n = 2, 3n + 4 = 3 \times 2 + 4 = 6 + 4 = 10$
- when $n = 3, 3n + 4 = 3 \times 3 + 4 = 9 + 4 = 13$
- when $n = 4, 3n + 4 = 3 \times 4 + 4 = 12 + 4 = 16$
- when $n = 5, 3n + 4 = 3 \times 5 + 4 = 15 + 4 = 19$

The first five terms of the sequence: $3n + 4$ are $7, 10, 13, 16, 19, \ldots$

Using the nth term to find if a number is in a sequence

Is the number 14 in the sequence 4n + 2?

$$4n + 2 = 14$$
$$-2 \qquad -2$$
$$4n = 12$$
$$\div 4 \qquad \div 4$$
$$n = 3$$

If you get a decimal here, then the term isn't in the sequence

Yes, 14 is the 3rd term in the sequence.

Online clips

M381, M241, M166, M991

Ratio

Key Vocabulary

Ratio	Relationship between two or more numbers.
Part	This is the numeric value '1' of, would be equivalent to
Simplify	Divide all parts of a ratio by the same number.
Equivalent	Equal in value

Ratio Notation

The ratio of apples to oranges is 3:2

Apples are mentioned first that is why the 3 comes before the 2

The ratio of oranges to apples is 2:3

Oranges are mentioned first that is why the 2 comes before the 3

To write this as a fraction

Apples to Oranges

3:2

There are 5 parts to this ratio in total

$$\frac{3}{5} : \frac{2}{5}$$

Simplifying a ratio

Simplify

12 and 20 both have a factor of 4

$\div 4$ 12:20 $\div 4$

3:5

Therefore both numbers need to be divided by 4

Using the highest common factor gets the final answer quicker but you could have divided by 2 and then 2 again for the example above

Simplify

$\div 10$ 60:40:100 $\div 10$

6:4:10

$\div 2$ $\div 2$

3:2:5

Alternatively, you could have divided by 20 here to simplify in one step

Writing ratios as 1:n or n:1

This means that the ratio needs to be simplified in a specific way. You may end up with fractions or decimals as part of your answer

Write 2: 5 in the form 1 : n

This means the left number needs to be 1

$\div 2$ 2 : 5 $\div 2$

1 : 2.5

Write 2: 5 in the form n : 1

This means the right numbers needs to be 1

$\div 2$ 2 : 5 $\div 2$

0.4 : 1

Sharing in a ratio

To share in a ratio, we can use bar modelling to visualise the steps.

Example

Share £35 in the ratio 2:5

There are 7 parts to this ratio

$35 \div 7 = 5$ so each part is worth £5

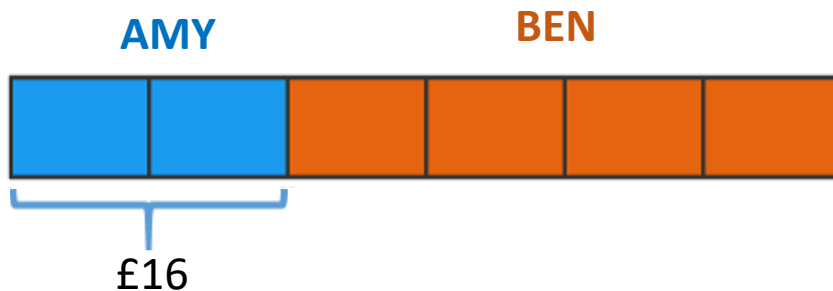| £5 | £5 | £5 | £5 | £5 | £5 | £5 |

10   25

£10 : £25

Sharing in a ratio when given one part

Example

Amy and Ben share some money in the ratio 2:4

Amy receives £16. How much does Ben get?

This example is different as Amy gets £16 they do not have £16 in total

AMY   **BEN**

£16

$£16 \div 2 = 8$

| £8 | £8 | £8 | £8 | £8 | £8 |

So Ben gets
$£8 \times 4 = £32$

Averages

Key Vocabulary

Data set	Collection of values that share a common relationship. This could be answers to a set question or information for a set objective.
Average	Is a value (or values) that is used to represent a whole data set
Mode	The most frequent value in a data set. It is a type of average. Modal is another word used more mode.
Median	The middle value of a data set, when ordered. It is a type of average.
Mean	A measure of the size of the data when shared out equally. It is a type of average.
Range	A value to show spread out a data set is. It can be used to describe how representative of the whole data set the average used is. IT IS NOT AN AVERAGE.

Averages

We use averages to summarise a whole data set in a single value/few values. We do this so we can interpret large data sets and also compare data sets more easily.

Mode- the most frequent value/ few values in a data set. There can also be no mode in a set of data.

Ex 1, find the mode:

blue   red   blue   green   blue   blue
pink   green   blue   red   blue   yellow   Blue is the mode.

Ex 2, find the mode:

9, 4, 3, 6, 9, 5, 2, 1, 8, 7

To make it easier, we can re-write these values in ascending(increasing) order.

1, 2, 3, 4, 5, 6, 7, 8, 9, 9. We can now see clearly **9 is the mode**.

Ex 3, find the mode:

9, 4, 3, 6, 9, 5, 2, 1, 8, 7, 3

Re-written 1, 2, 3, 3, 4, 5, 6, 7, 8, 9, 9 We can see **3 and 9 are the modal values**.

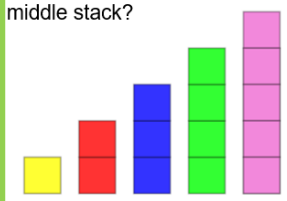** We usually only have 1, 2 or 3 modal values**

Ex 4, find the mode:

4, 3, 6, 9, 5, 2, 1, 8, 7

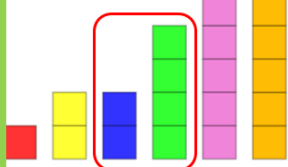Re-written 1, 2, 3, 4, 5, 6, 7, 8, 9 We can see there **are NO modal values**.

Median- the middle value in a data set, when in order. If there are 2 middle values, we find the midpoint between them.
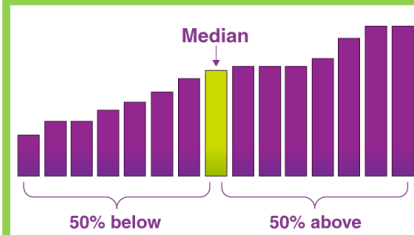
How many blocks are in the middle stack?

The middle stack has 3 blocks in

How many blocks are in the middle stack?

There is no "middle stack". We have to calculate the middle of 2 and 4. The middle would be 3.

Median

50% below   50% above

Find the median of: 1, 3, 3, **6**, 7, 8, 9

Median = 6

Find the median of: 1, 2, 3, **4, 5**, 6, 8, 9

Median is the midpoint of 4 and 5 = 4.5

Find the median of the following set of numbers.
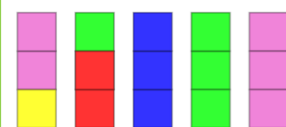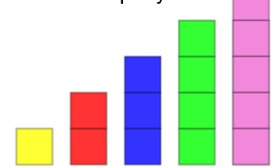
40   −2   10   40   −31   3   −34   −13   −10   1   30   16   −16

−34   −31   −16   −13   −10   −2   1   3   10   16   30   40   40

Mean- The mean is the size of each part when a quantity is shared equally. We can do this by adding all the values in the data set together and then dividing it equally between the number of values.

How many blocks would there be in each stack if they were shared out equally?

There would be three in each pile so the mean = 3

Example 1.
Find the mean of the following set of numbers.

19, 6, 17, 6

Solution.
To find the mean divide the sum of the numbers by the number of numbers.

$$\frac{\text{Sum of numbers}}{\text{Number of numbers}} = \frac{19+6+17+6}{4}$$

$$= \frac{48}{4}$$

$$= 12$$

There are 4 values in the data set so we are dividing by 4.

Range- the range shows how spread out the data is. It is useful to order the data when finding the range. The smaller the range, the more consistent the data.

E.g. Find the range of the following numbers

43   36   10   −8   −3   −6   −4   −22

−22   −8   −6   −4   −3   10   36   43

65

Range =43- -22 = 65

Online Clips

M841, M934, M940, M328

Pictograms

Component Knowledge

- To be able to draw a pictogram given a frequency table
- To be able to interpret a pictogram.

Key Vocabulary

Pictogram	A type of chart to represent non – numerical or discrete data
Key	Shows what value each picture represents.
Discrete	Data that can only take certain fixed values – it usually is values that can be counted. (e.g. whole numbers, shoe sizes, money)
Frequency	The number of times the item occurs in an experiment.

A **pictogram** uses pictures or icons to represent the total frequency of each category.

In a pictogram the icons used must be the same size and equally spaced. This makes it very easy to spot the mode (most common).

A key is included to indicate what value each icon represents. Fractions of the icon, usually a half or a quarter, might be used to indicate some values.

A pictogram can be used to make comparisons between the categories in a set of data.

Drawing

1. Look at the largest frequency in your table
2. Decide on an appropriate icon and what value it will represent, ones twos or fours work best.
3. Draw a table with two columns, one for category and one for the icons.
4. Label the categories in the table.
5. Draw the correct number of images for each frequency.
6. Draw the correct number of images for each frequency.
7. Check you have included a key and give your pictogram a title.

Soft drink	Apple	Orange	Blackcurrant	Other
Frequency	8	10	3	7

A pictogram shows students' favourite soft drink — KEY = 2 students

Interpreting

The pictogram shows the number of driving lessons some students had during a month. If Simon had 7 lessons, how many lessons did Samaira have?

A pictogram to show the number of driving lessons during a month — KEY = ? lessons

Simon has three and a half Icons. This represents seven lessons (7 ÷ 3.5 = 2)

Each icon represents two lessons. Samira has four and a half icons. Samira = 4.5 x 2 = 9 lessons.

Online clip

M644

Bar Charts

Key Vocabulary

Tally chart	Table used to track the frequency of a variable
Frequency	Measures how often things are repeated
Height	Measure from the base to the top of an object
Width	Measure of how wide an object is, usually horizontally
Axis	Horizontal or vertical scale on a graph or chart
Axes	Plural of axis, i.e. a set of axes (pronounced 'axees')
Discrete	Data that can only take certain values, e.g. shoe size
Continuous	Data that are not fixed and can have an infinite set of values

Completing a Tally Chart

Complete a tally chart for the most popular colour of car:

Red, blue, red, green, red, purple, red, green, red, purple, green, blue, red, green, blue, red, red, red

For each colour, draw a **vertical line** in the 'tally' column.

For the 5th number, you put a slash through the bundle of 4 vertical lines

Colour	Tally	Frequency
Red	̶I̶I̶I̶I IIII	9
Blue	III	3
Green	IIII	4
Purple	II	2

Add the tally to give a **numerical value**. This is called the **frequency**

Key facts & Tips

Bar charts represent *discrete data* (i.e. data you can count).
The bars are **equal width** with **equal gaps** between them. The height of each bar represents the frequency, which is shown on the y-axis

EXAM TIP
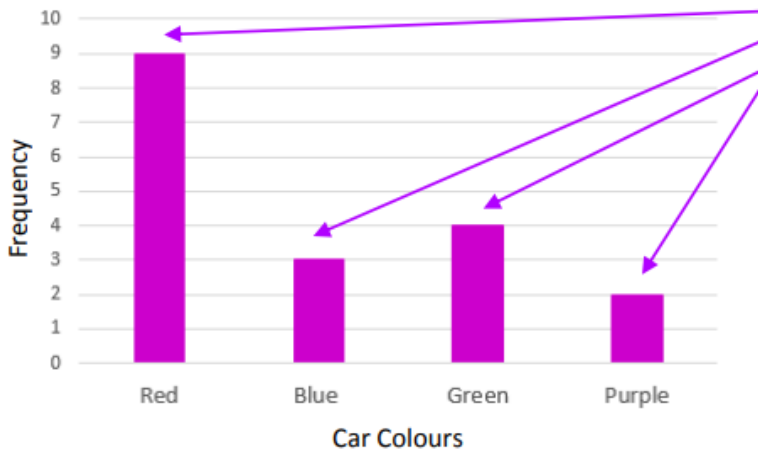
Don't forget to label the graph

Example Question

Mrs Klingon asks 21 of her colleagues what model of car they drive. Below is the data.

Car Model	Frequency
Ford	6
VW	3
Kia	11
Nissan	1

Draw a bar chart to represent this.

Completing the bar chart

The number of red, blue, green and purple cars is the **frequency (height of the bars)**.

IMPORTANT

The **bars** are the SAME width

The **gaps** between the bars are the SAME width

Both axes are **labelled**

The graph has a **title**

Frequency starts at 0

Further example question & solution

1) **The table shows the number of year 11 students who attended intervention in one week.** Use this information to draw a bar chart.

Mon	Tues	Wed	Thurs	Fri
12	15	3	7	1

2) **The data below shows the favourite fast food of a group of year 7 students. Use this information to complete a tally chart. Then draw a bar chart to represent the information.**

Pizza, burger, pizza, hotdog, burger, pizza, burger, pizza, hotdog, pizza, pizza, hotdog, pizza, burger pizza, burger, hotdog, chicken wings, burger
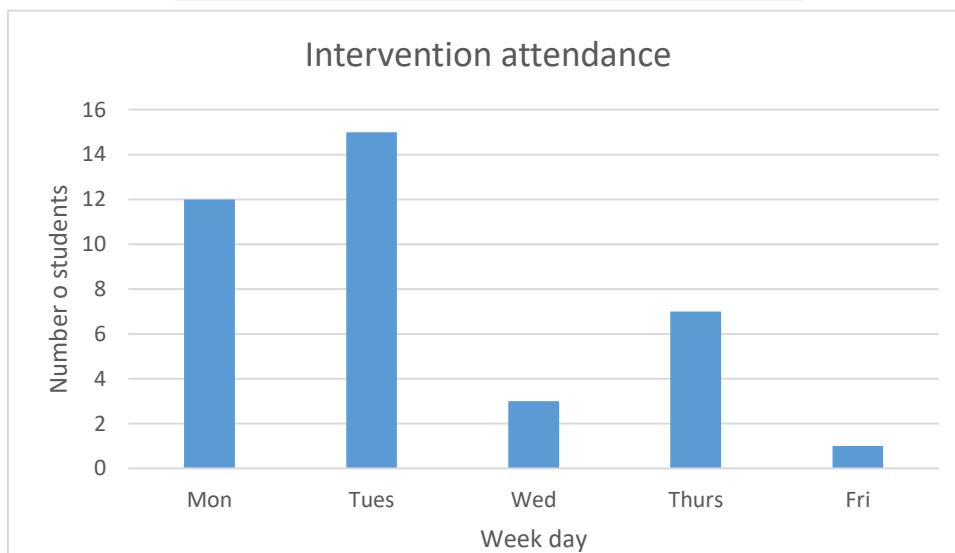
Online clips

M460, M738

Line Graphs

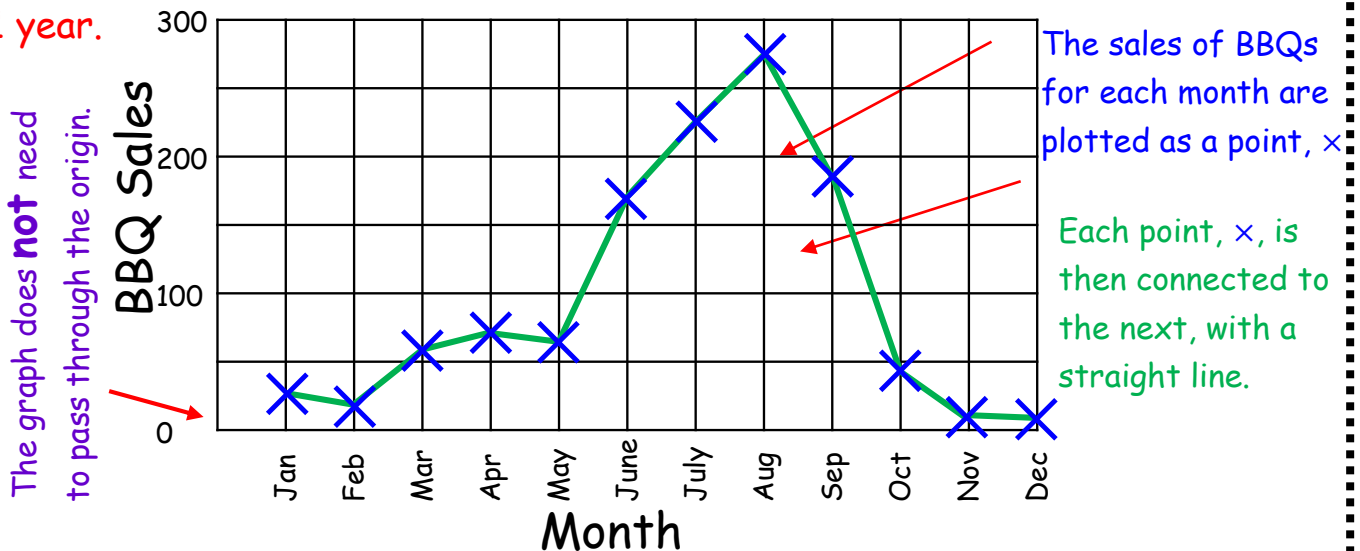## Key Vocabulary

Series	A set of sequential pieces of data
Trend	A pattern in a data series

Line Graphs (Time Series Graphs):

Are commonly used to show how a variable changes over *time*.
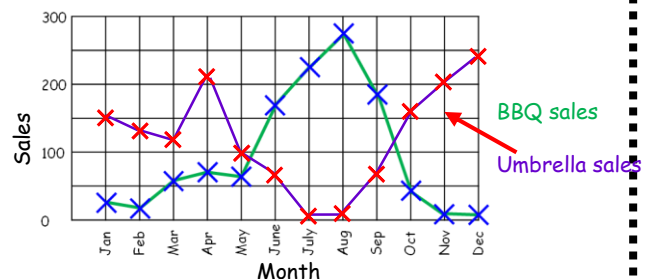
The graph is used to show *trends* in data.

e.g. A graph to show sales of BBQ's in a shop each month, over the course of 1 year.

The graph does **not** need to pass through the origin.

The sales of BBQs for each month are plotted as a point, ×

Each point, ×, is then connected to the next, with a straight line.

Here sales of BBQ's increase from May to August before decreasing again.

A second data series can be plotted on the same axes to allow *comparison*.

Here the highest months for sales of BBQs are July and August, these are the lowest months for sales of umbrellas.

BBQ sales

Umbrella sales

Online clips

M140, M183