Cancellation to simplify

Component Knowledge

- To be able to simplify fractions using highest common factors

Key Vocabulary

Fraction	A fraction is made up of a numerator (top) and a denominator (bottom).
Equivalence	Two fractions are equivalent if one is a multiple of the other.
Simplify	Cancel a fraction down to give the smallest numbers possible.

Cancelling to simplify

If a numerator and denominator share a multiplication factor they can be cancelled

Example

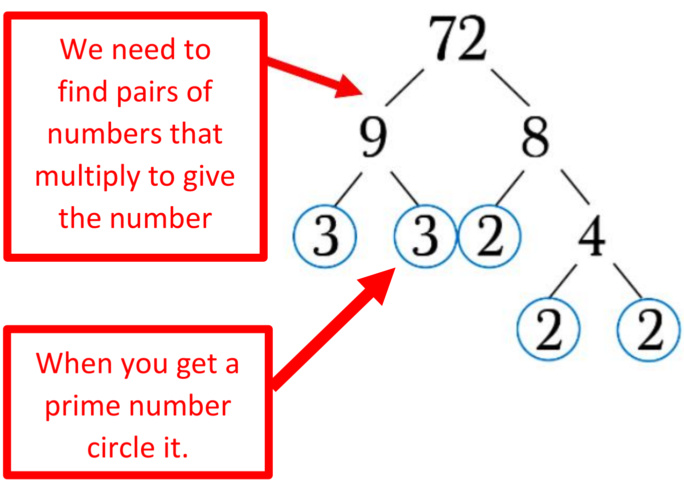
$$\frac{2\times \cancel{3}\times \cancel{3}}{3\times \cancel{3}\times \cancel{3}}$$
$$=\frac{2}{3}$$

Prime Factors

To be able to cancel the factors it helps to write your numerator as a product of its prime factors

Reminder:

Write 72 as a product of its prime factors



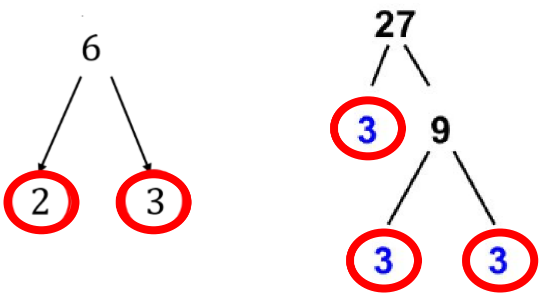
Online clips

Example

Simplify

$$\frac{6}{27}$$

First write the numerator and denominator as a product of their prime factors.



$$\frac{6}{27}=\frac{2\times \cancel{3}}{\cancel{3}\times 3\times 3}$$
$$=\frac{2}{3\times 3}=\frac{2}{9}$$

Fractions of Amounts

Component Knowledge

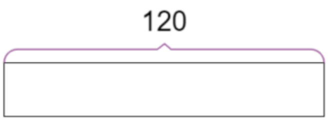
- To calculate fractions of amounts

Key Vocabulary

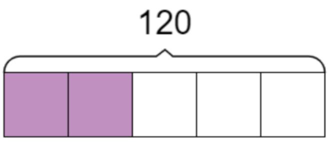
Fraction	A way of writing a part of an integer(whole number).
Numerator	The top number in a fraction- the number of parts of the whole we have/want.
Denominator	The number of equal parts the whole has been divided into equally.
Of	Means parts of or multiply.

Fractions of Amounts- non-calculator

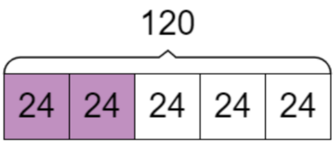
$Find\ \frac{2}{5}\ of\ 120$



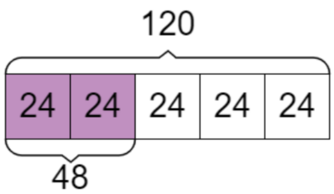
Draw a bar model



Shade $\frac{2}{5}$ of the bar



Divide 120 (amount) by 5 (number of parts) = 24



Two parts equal 2 x 24 = 48

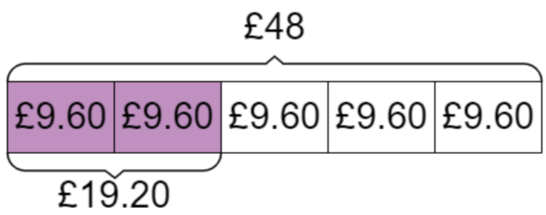
Fractions of Amounts-Money

$Find\ \frac{2}{5}\ of\ £48$

48÷5 =9.6

Remember money is shown to 2dp

so, 9.6 = £9.60

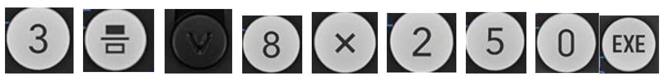


Fractions of Amounts- calculator

$Find\ \frac{3}{8}\ of\ £250$

Of means multiply so swap the of to ×

$Type\ \frac{3}{8} \times\ £250\ into\ your\ calculator$



Answer = £93.75

Online clips

M695, M684

Equivalent fractions

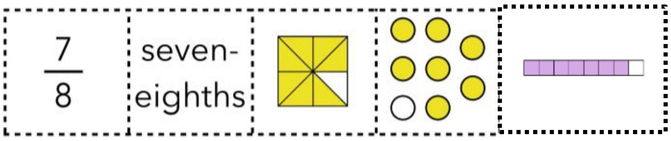
Component Knowledge

- To understand fractions are part of a whole.
- To be able to calculate equivalent fractions
- To use equivalent fractions to compare the size of fractions.

Key Vocabulary

Fraction	A way of writing a part of an integer(whole number).
Numerator	The top number in a fraction- the number of parts of the whole we have/want.
Denominator	The number of equal parts the whole has been divided into equally.
Equivalent	Means equal to.

Fractions- can be written numerically or as diagrams.



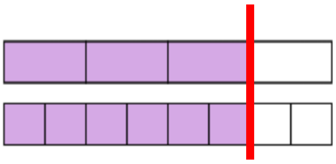
$\frac{1}{2}$ *means 1 part out of 2 parts of the whole*

$\frac{7}{8}$ *means 7 parts out of 8 parts*

Number of parts you have/want → *numerator*

denominator ← Number of equal parts in total

Equivalence- some fractions are equal in size, even when they look different.



The bars show $\frac{3}{4} = \frac{6}{8}$. You can see they have the same size, even though the parts in the bars are different sizes.

To calculate equivalent fractions, we need to multiply or divide by a common number.

Find $\frac{2}{5} = \frac{}{20}$

We need to find the number we multiply 5 by to get the answer of 20. This is 4 (5 x 4=20).

$\frac{2}{5} \times \frac{4}{4} = \frac{8}{20}$ **So,** $\frac{2}{5} = \frac{8}{20}$.

Comparing- to compare fractions, we need all fractions to have the same denominator.

Same denominator—compare the numerators

$\frac{2}{8} < \frac{5}{8}$

Change the denominator of one to match the other

$\frac{2}{5}$ and $\frac{3}{10}$ $\quad \frac{2}{5} \times \frac{2}{2} = \frac{4}{10} \quad \frac{2}{5} > \frac{3}{10}$

Change both denominators to a common denominator

$\frac{7}{8}$ and $\frac{5}{6}$ $\quad \frac{7}{8} \times \frac{3}{3} = \frac{21}{24} \quad \frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

$\frac{7}{8} > \frac{5}{6}$

Online clips M410, M671, M335

Four operations with fractions

Component Knowledge

- To be able to convert between mixed numbers and improper fractions
- To be able to use equivalent fractions
- To be able to add and subtract fractions including mixed numbers
- To be able to multiply fractions
- To be able to divide fractions.

Key Vocabulary

Numerator	The top part of a fraction – how many parts are represented.
Denominator	The bottom part of a fraction – This tells us how many parts there are in the whole.
Equivalent	Two fractions are equivalent if one is a multiple of the other. They have equal value.
Mixed number	Are made up of a whole number (integer) and a fraction.
Improper fraction	A fraction where the numerator is larger than the denominator.
Reciprocal	The reciprocal of a number is 1 divided by the number. When we multiply a number by its reciprocal, we get 1. This is why it is called the multiplicative inverse. E.g the reciprocal of 2/3 is 3/2.
Simplify	To cancel down a fraction to give the smallest possible numbers. We do this by dividing the numerator and the denominator by the highest common factor.

Improper fraction to mixed number

$$4\frac{3}{5}$$

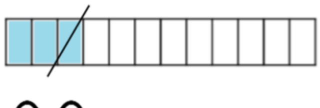
$$= \frac{4 \times 5 + 3}{5}$$

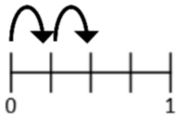
Multiply the denominator by the whole number then add the numerator

$$= \frac{23}{5}$$

Add/Subtract unit fractions

Same denominator

$$\frac{1}{12} + \frac{1}{12} - \frac{1}{12}$$  $$= \frac{2}{12}$$

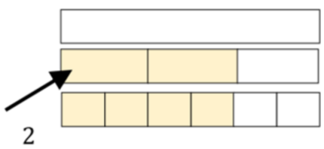
$$\frac{1}{4} + \frac{1}{4}$$  $$= \frac{2}{4}$$

With the same denominator ONLY the numerator is added or subtracted

Equivalent fractions

Numerator and denominator have the same multiplier

$$\frac{2}{3} = \frac{4}{6}$$



$$\frac{1}{3} = \frac{2}{6}$$

Adding Fractions

Example: $\frac{3}{5} + \frac{2}{7}$

×7 ×5

$$\frac{21}{35} + \frac{10}{35} = \frac{31}{35}$$

To add fractions the denominators must be the same. First choose the lowest common multiple of both denominators to be the new denominator.
Then use equivalent fractions to keep the sum the same.
Then add the numerators as with unit fractions.

Adding mixed numbers

Add the following fraction, give your answer in its simplest form:

$$5\frac{1}{8} + 3\frac{5}{6} = 5\frac{3}{24} + 3\frac{20}{24}$$

Find a common denominator.

$$= 8 + \frac{23}{24}$$

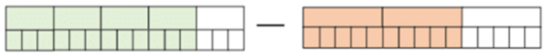
Add the integers, and then add the fractions.

$$= 8\frac{23}{24}$$

Add.

Subtracting Fractions

$\frac{4}{5} - \frac{2}{3}$

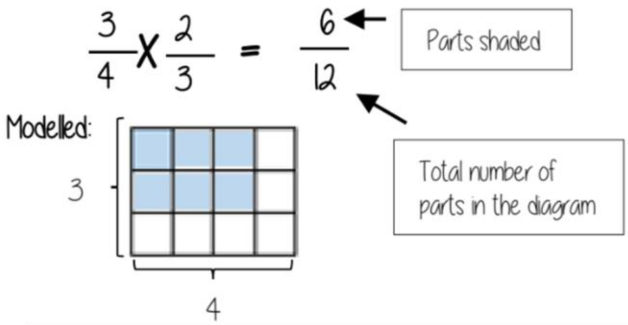


$= \frac{2}{15}$

$\frac{12}{15} - \frac{10}{15}$

Use equivalent fractions to find a common multiple for both denominators

Multiplying Fractions



To multiply two fractions: Multiply the numerators. Multiply the denominators.

If using mixed numbers convert to improper fractions first.

Subtracting mixed numbers

$2\frac{1}{5} - 1\frac{3}{10}$

$2\frac{2}{10}$

$\frac{22}{10} - \frac{13}{10} = \frac{9}{10}$

- Use equivalent fractions to find common denominators.
- Change to improper fractions
- Subtract the numerators.
- If needed simplify

Dividing Fractions

When dividing fractions we use reciprocals. To find the reciprocal we 'flip' the fraction. (It is the multiplicative inverse of the fraction).

E.g. the reciprocal of 3 is $\frac{1}{3}$, the reciprocal of $\frac{1}{6}$ is 6 etc, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$ etc

We multiply by the reciprocal of the second fraction.

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiply by a reciprocal gives the same outcome

We can use KFC to help us remember the method.

- Keep the first fraction the same
- Flip the second fraction (convert to its reciprocal)
- Change the divide to a multiply (as we are using the multiplicative inverse).

Remember to convert to improper fractions when using mixed numbers.

Online clips

M410, M671, M835, M931, M157, M197,

M216, M110, M265, M645, M619

Algebraic Vocabulary

Component Knowledge

- Understand the difference between the various algebraic words
- Understand how each previous word builds on to the next

Key Vocabulary

Variable	A quantity that can take on many values denoted by a symbol or a letter
Term	Is a single variable or number or variables and numbers multiplied together.
Expression	A group of numbers, letters and operational symbols, e.g. 2x + 3y -8
Equation	A number statement with an equals sign (=). Expressions on either side of the equals sign are of equal value, e.g. a + 14 = 20 or 2(x + 12) = 44 or x + 5 = 2x + 3
Formula	A special type of equation that shows the relationship between different variables. They tend to describe real-world situations. Plural is formulae.
Identity	An equation where both sides are identical whatever the value of the variable

A **variable** is a symbol (often a letter) that is used to represent an unknown.

E.g. x or y or a etc.

Variables can also have exponents (can be raised to a certain power.

E.g. x^2

A coefficient is the value that is before a variable. It tells us how many lots of the variable there is.

E.g. $x + x + x + x + x = 5 \times x = 5x$

The coefficient here is 5.

An **algebraic term** is either a single number or a variable.

e.g. '3' or 'x' or 'h'

A term can also be a number and a variable multiplied together.

e.g. 2a or 6y or 4xy

When 2 or more algebraic terms are added (or subtracted) they form an expression.

Formula/Formulae

A formula is a special type of equation that shows the relationship between different substituted variables. Formulas are often used in geometry to find area and volume.

Area of triangle = (base × height) ÷ 2

Area of rectangle = length × width

(12.5 × hours worked) + 25 = cost of job

Algebraic identities use the '$\equiv$' symbol. It is like an equal's sign, but it means identical to. No matter what the value of the variable this will always be true. e.g. 2x = x + x

An **algebraic expression** is a single term or a set of terms that are combined using addition (+), subtraction (-), multiplication (x) and division (÷)

Examples

$3x$ $\qquad 2x + 3y$ $\qquad 2 - 5y^2$ $\qquad 2x + 3y - 5$

An expression that contains two terms is called a binomial.

Equations are mathematical expressions which contain one or more variables and an equals sign.

Examples $\quad 3x - 5 = 7 \qquad \frac{4(x-2)}{5} = 8 \qquad x^2 = 9 \qquad 2x^2 - 3x - 5 = 0$

We can solve an equation to find the value of the variable(s).

Example Solve $4x + 3 = 23$

$$4x + 3 = 23$$
$$-3 \qquad -3$$
$$4x = 20$$
$$\div 4 \qquad \div 4$$
$$x = 5$$

Online clips

M813, M830

Collecting Like terms

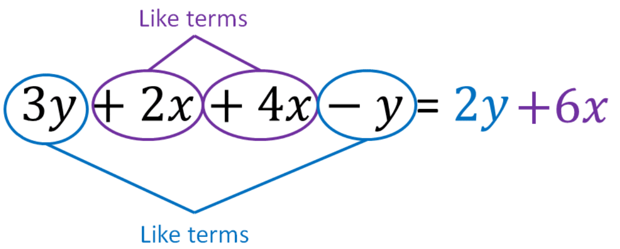
Component Knowledge

- Recognise terms in algebra
- Use of positive and negative directed numbers

Key Vocabulary

Variable	A **Variable** is a symbol for a number we don't know yet. It is usually a letter like x or y
Term	A **Term** is either a single number or a variable (x), or numbers and variables multiplied together ($5y$).
Expression	An **Expression** is a group of terms (the terms are separated by + or − signs) (eg, $5y + 6x - 8y$)
Simplify	reducing the expression/fraction/problem in a simpler form.

Collecting like terms : We collect like terms to simplify an expression. We look at terms which share the same variable



In this example:

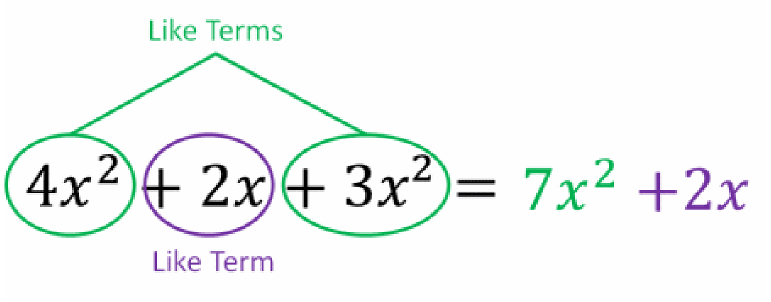
We collect all the x variables : $2x+4x = 6x$

AND

Collect all the y variables: variables : $3y-y = 2y$

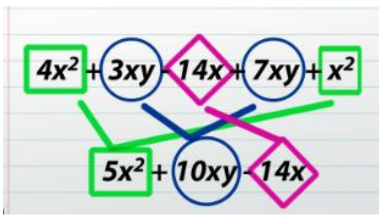
Collecting like terms – example 2

When collecting like terms, it is important to find the same terms and combine them to simplify the algebraic expression. We need to be able to recognise that x is different to x^2



Handy Hint:

It helps if you can visually see the different terms before you collect them. Using a different coloured pen, highlighter or shape works!



Online Clips

M795, M531, M949

Simplifying Expressions

Component Knowledge

- Law of indices
- Collecting like terms
- Recognise Algebraic terms and expressions

Key Vocabulary

Terms	In Algebra a term is either a single number or variable
Expression	Numbers, symbols and operators grouped together to show the value of something
Simplify	Reducing the expression/fraction to a simpler form.

Simplifying Terms - Multiplying:

Algebraic terms can be multiplied to give a simplified term. We focus on the number first, and then the variable (x or y), often using laws of indices.

Important – we always write terms in alphabetical order

Example	Answer
$2x \times 3 =$	$6x$
$4a \times 5b =$	$20ab$
$y^2 \times y^3 =$	$y \times y \times y \times y \times y = y^5$
$2ab \times 8cd =$	$2 \times 8 \times a \times b \times c \times d = 16abcd$
$a^5 b^3 \times a^4 bc^2 =$	$a^9 b^4 c^2$

Remember, any number to the power 0 is always 1

Simplifying Terms - Dividing:

Algebraic terms can be divided to give a simplified term. We focus on the number first, and then the variable (x or y), often using laws of indices.

Important – we should always write the division as a fraction,

e.g. $12a \div 6 = \frac{12a}{6}$

Example	Answer
$\frac{12a}{6} =$	$2a$
$\frac{18x}{24} =$	$\frac{3x}{4}$
$y^5 \div y^3 =$	$\frac{y \times y \times y \times y \times y}{y \times y \times y} = y^2$
$15a^4 \div 3a^2 =$	$\frac{15 \times a \times a \times a \times a}{3 \times a \times a} = 5a^2$
$a^3 \div a^3 =$	1

Online Clips

M795, M531, M120

Forming Expressions and Equations

Component Knowledge

- To be able to form expressions and equations from worded problems.

Key Vocabulary

Expression	A mathematical statement written using symbols, numbers or letters
Equation	A statement showing that two expressions are equal.
Variable	A symbol representing an unknown value
Substitute	To replace a variable with a given value
Simplify	To write an expression in its most efficient way without changing the value of the expression.
Solve	Find the of the unknown that makes the equation true
Form	Bring together parts or combine to create something

Writing expressions

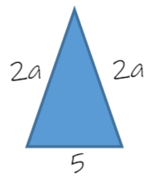
We can use algebra to express values which are unknown to us

e.g. 2 more than w would be w + 2

3 lots of w would be 3w

5 fewer than w would be w – 5

We can also use it to write formulas or expressions for shapes e.g. the perimeter of this triangle is 4a + 5



Creating Expressions using Function Machines

Input	Operation	Output
x	×2	$2x$
x	÷6	$\frac{x}{6}$
e	+5	$e + 5$
x	−7	$x - 7$

Input	Operation 1	Operation 2	Output
x	×2	+5	$2x + 5$
x	÷3	+7	$\frac{x}{3} + 7$
x	−2	÷4	$\frac{x-2}{4}$
x	+5	−2	$x + 3$
x	×2	×3	$6x$

Set up equations from word problems

Jenny, Kenny, and Penny together have 51 marbles. Kenny has double as many marbles as Jenny has, and Penny has 12. How many does Jenny have?

Set up an equation then solve

Jenny's + Kenny's + Penny's = 51

n + 2n + 12 = 51

3n + 12 = 51

-12 -12

3n = 39

÷3 ÷3

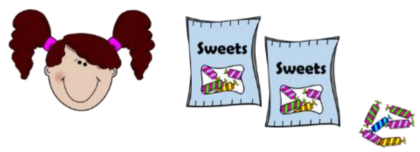
n = 13

Start by writing your first unknown value as a variable e.g. n

Expressions from Worded Problems

Cindy has 2 bags of sweets and 6 loose sweets.

How many sweets does she have?



We don't know how many sweets are in a bag.

So we will **express** is using a **letter** instead.

b = the number of sweets in a bag.

$$2b + 6$$

Online clip

M957

Expanding single brackets

Component Knowledge

To be able to expand a single bracket, including problems with powers.

Key Vocabulary

Expression	A mathematical statement written using symbols, numbers or letters.
Simplify	In general, an expression is in simplest form when it is easiest to use
Expand	Expand is when we multiply to remove the ( )

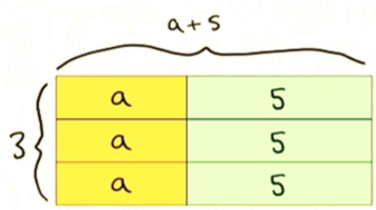
Expanding brackets means multiplying everything inside the bracket by the letter or number outside the bracket.
For example, in the expression $3(m+7)$ both m and 7 must be multiplied by 3:
$3(m+7)$
$=3 \times m + 3 \times 7$
$=3m + 21$

Expanding brackets involves using the skills of simplifying algebra. Remember that $2 \times a = 2a$

Example
Expand $4(3n+y)$.
$=4 \times 3n + 4 \times y$
$= 12n + 4y$

Using grid method

Expand: 3(a + 5)   3 x (a + 5)



3 x a = 3a   3 x 5 = 15

3a + 15

Using arrows

Expand:

$7(3 + a) = 21 + 7a$

$3x(5x + 2) = 15x^2 + 6x$

Expanding and simplifying

To expand and simplify more than one bracket, first expand each bracket then collect like terms.

$2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a$
$= 5a + 16$

Note – collect like terms to simplify

$4(x + 2) - 2(x + 2) = 4x + 8 - 2x - 4$
$= 2x + 4$

Note: Remember the rules when multiplying negatives, - 2 multiplied by x = -2x

Online clips

M237, M792

Factorise single brackets

Component Knowledge

- To be able to factorise into a single bracket with a numerical common factor.
- To be able to factorise into a single bracket with a variable as a common factor.
- To be able to factorise expressions involving powers into a single bracket.

Key Vocabulary

Factorise	Putting an expression back into brackets
Brackets	Symbols used in pairs to group things together
Term	A single number, variable or numbers and variable multiplied together
HCF	Highest common factor

Factorise a single bracket numerical factor

Factorising to a single bracket means that we take out the **highest common factor** from each term in an algebraic expression, and then write the expression as a **product** of the HCF and a single bracket.

Example

$3x + 6 = 3(x + 2)$

3 is the HCF of $3x$ and 6, so this is written outside the single bracket.

Example

$14x - 21 = 7(2x - 3)$

7 is the HCF of 14x and 21, so is written outside the bracket.
$7 \times 2x = 14x$,
$7 \times -3 = -21$

Factorise a single bracket with variables as factors

In this example there are no numerical factors but x is a factor (as $x^2 = x \times x$)

Factorise $x^2 + 4x$.

Find the HCF of the terms	$x^2 + 4x$	**HCF** $= x$
Write the HCF and 'open' the brackets	$= x(\quad\quad)$	
Divide each term by the HCF to find the values inside the bracket.	$= x(x + 4)$	

This example has numbers and variables as factors.

Factorise $6x + 3x^2$.

Find the HCF of the terms	$6x + 3x^2$	**HCF** $= 3x$
Write the HCF and 'open' the brackets	$= 3x(\quad\quad)$	
Divide each term by the HCF to find the values inside the bracket.	$= 3x(2 + x)$	

Online clip

M100

Substitution

Component Knowledge

- To substitute positive and negative numbers into expressions with one, or more, variables.

Key Vocabulary

Expression	A maths sentence that includes a minimum of 2 variables, including an algebraic term and at least one operation.
Term	Either a single number or variable, or the product of several numbers or variables.
Substitute	To exchange an unknown variable for a number in an expression/equation/formula.

Substitution-formula

For example: The time in minutes to cook a chicken is given by the formula:

Time = 40 minutes per kilogram plus 20 minutes

Find how long it takes to cook a 5kg chicken.

Here we substitute 5kg into the formula. **So, Time= 40 x 5 +20 = 220 minutes**

The formula for speed is shown: $Speed = \frac{Distance}{Time}$

Find the average speed when travelling 150 miles in 4 hours.

Here we substitute Distance = 150 and Time = 4 into the formula. $Speed = \frac{150}{4} = 37.5mph$

Substitution-expressions

Example 1

$f = p + 4$. find the value of f when $p = 6$.

We substitute 6 for p in the formula.

$f = (6) + 4$

f = 10

Example 2

$f = 2p + 4$. find the value of f when $p = -6$.

We substitute -6 for p in the formula.

$f = 2(-6) + 4$

f = -8

Example 3

$f = t^2$. find the value of f when t = -6.

We substitute -6 for t in the formula.

$f = (-6)^2$

f = 36

Example 4

$f = \frac{t^2}{5y}$. find the value of f when t = -6, y = 4.2

We substitute -6 for t and 4.2 for y in the formula.

$f = \frac{(-6)^2}{5(2.4)}$

$f = \frac{36}{12}$

When substitute negative numbers, we must put brackets around them to ensure the correct order of operations occurs. **This very important when we use calculators.** (We can also do this with positive numbers)

From example 4. $-6^2 = -(6)^2 = -36$ is not equal to $(-6)^2 = -6 \times -6 = 36$.

Online clips: M417, M327, M208, M979

Function machines and solving 1 and 2 step equations

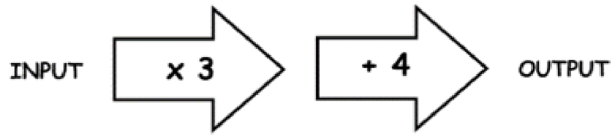
Component Knowledge

- To be able to use function machines to find the input and output value.
- To be able to solve one-step equations.
- To be able to solve two-step equations.

Key Vocabulary

Function Machine	Takes an input value, performs some operations and produces an output value.
Operation	Common operations are addition, subtraction, multiplication and division.
Inverse	The operation of another function.
Equation	a mathematical statement that shows that two mathematical expressions are equal
Solve	To find the solution

Function Machines



If the input is 5 the calculation is

5 x 3 = 15

15 + 4 = 19

→ [-3] → [x6] → 12

5 ← [+3] ← [÷6] ← 12

To find the input, start with the input and work backwards doing the inverse operations of the function machine.

One- step equations

To solve a one-step equation, you need to do the inverse operation.

÷5: $5x = 30$ → $x = 6$ ÷5

+3: $x - 3 = 7$ → $x = 10$ +3

-5: $x + 5 = 9$ → $x = 4$ -5

The inverse of multiplying is dividing.

We divide 30 by 5.

The inverse of subtracting is addition.

We add 3 to 7.

The inverse of addition is subtraction.

We subtract 4 from 9.

x 3: $\frac{x}{2} = 3$ → $x = 6$ x 3

The inverse of dividing is multiplying.

We multiply 2 by 3.

Two- step equations

To solve a one-step equation, you need to do the inverse operation.

To solve a two-step equation or inequality we need to complete 2 inverse calculations in a specific order.

$6x + 3 = 32$

-3 — The inverse of adding 3 is subtracting 3

$6x = 30$

÷ 6 — The inverse of multiplying 6 is dividing by 6

$x = 5$

$4x - 3 = 13$

+3 — The inverse of subtracting 3 is adding 3

$4x = 16$

÷ 4 — The inverse of multiplying 4 is dividing by 7

$x = 4$

$\frac{x-5}{3} = 4$

÷ 3 — The inverse of dividing by 3 is multiplying by 3

$x - 5 = 12$

+ 5 — The inverse of subtracting 5 is adding 5

$x = 17$

Online clips

M175, M428, M707, M634, M647, M855, M401