Place Value

Component Knowledge

- Identify the value of digits within a number
- Write a number in words
- Write a value in figures from words

Key Vocabulary

Digit	The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 & 9 used to build numbers
Number	The value of a single/string of digits derived from their position in the string
Place value	The relative value of each position in a number
Place value table	A table with columns for each value to allow
Integer	Whole numbers

Place value table:

Integer values: "Write in words the number 32 406 059"

Place Value

Trillions			Billions			Millions			Thousands			Units			Decimals			
Hundred Trillion	Ten Trillion	Trillion	Hundred Billion	Ten Billion	Billion	Hundred Million	Ten Million	Million	Hundred Thousand	Ten Thousand	Thousand	Hundred	Ten	Unit	Tenths, $\frac{1}{10}$	Hundredth, $\frac{1}{100}$	Thousandth, $\frac{1}{1000}$	Ten thousandths ...
							3	2	4	0	6	0	5	9				...
																		...

In words: Thirty-two million, four hundred and six thousand and fifty-nine

Fractional values: "Write the value of the 4 in the number 27.104"

Place Value

Trillions			Billions			Millions			Thousands			Units			Decimals			
Hundred Trillion	Ten Trillion	Trillion	Hundred Billion	Ten Billion	Billion	Hundred Million	Ten Million	Million	Hundred Thousand	Ten Thousand	Thousand	Hundred	Ten	Unit	Tenths, $\frac{1}{10}$	Hundredth, $\frac{1}{100}$	Thousandth, $\frac{1}{1000}$	Ten thousandths ...
													2	7	1	0	4	...
																		...

Value: Four thousandths

Online clips

M763, M704, M522

Powers of 10

Component Knowledge

- Multiply and divide by powers of 10
- Understand what a square and a cube number is

Key Vocabulary

Index	The index of a number says how many times to use the number in a multiplication
Power	Another word for an 'index'. These include square/cube

Powers of 10 : We can use index form to write powers of 10 to a positive power.

$10000 = 10 \times 10 \times 10 \times 10$

$= 10^4$ ← We are multiplying 10 by itself 4 times

$100 = 10 \times 10$

$= 10^2$ ← We are multiplying 10 by itself 2 times or '10 squared'.

We can also use index form to write powers of 10 to a negative power

$\frac{1}{10} = 10^{-1}$ ← We are diving by 10

$\frac{1}{1000} = 10^{-3}$ ← We are diving by 10x10x10 or we are diving by 10^3

Powers of 10 and calculations

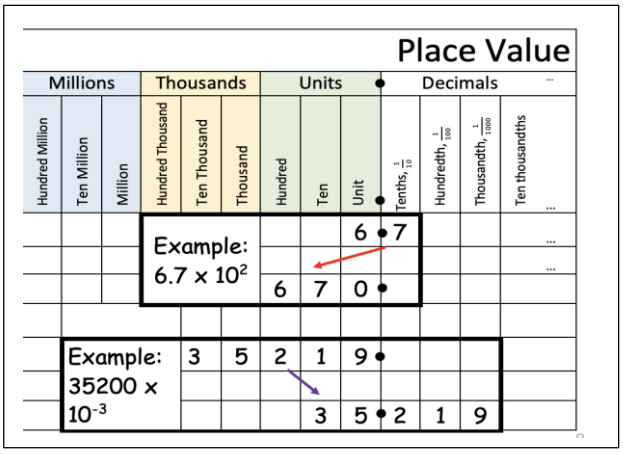
Powers of 10 : Using place value we know the value of each column is ten times greater than the column to the right.

Multiplying by 10, means the number is ten times greater, and moves one column to the left

Example:

$6.7 \times 10^2 = 670$

This means that 6.7 is 10 times and then 10 times bigger (or moves **two** columns to the



Example:

$35219 \times 10^{-3} = 35.219$

This means that 35219 is 10 times, then 10 times and then 10 times smaller (or moves **three** columns to the right)

Online Clip

M113

Ordering Positive and Negative Numbers

Component Knowledge

- Use a number line to order positive and negative numbers.
- Understand a number line is symmetrical about zero.

Key Vocabulary

Positive number	A number with a value greater than zero.
Negative number	A number with a value less than zero.
Ascending order	To list numbers from lowest value to highest value.
Descending order	To list numbers from highest value to lowest value.

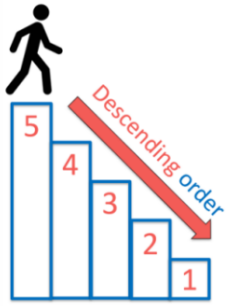
Ascending

smallest to largest

Ascending means going up

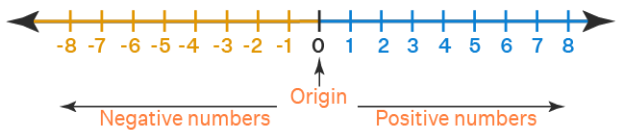
Descending

largest to smallest



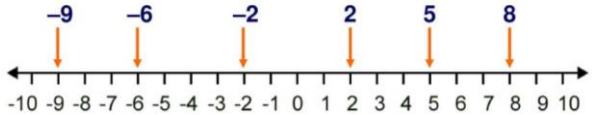
Descending means going down

Number lines can help us place numbers correctly so we can order them. They may be horizontal like this.



Write the integers –2, 8, 2, –6, –9 and 5 in order from smallest to largest.

Look at the position of the integers on the number line:

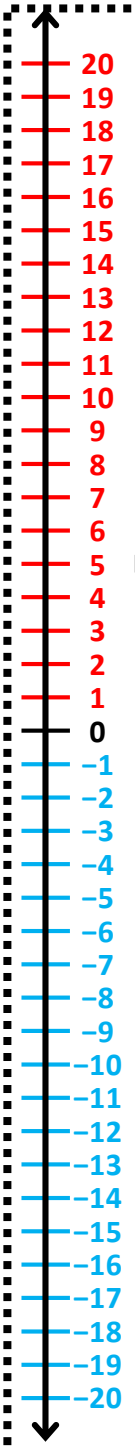


So, the integers in order are:

–9, –6, –2, 2 , 5, and 8

Online clips

Q976, M527



Some number lines can be vertical like this one They are helpful when ordering temperatures or heights.

Prague	–4°C
Lisbon	5°C
Vienna	–2°C
Helsinki	–5°C
Berlin	3°C
Manchester	14°C

List these cities from lowest temperature to highest.

The answer is

Helsinki, Prague, Vienna, Berlin, Lisbon, Manchester.

Adding & Subtracting Directed Number

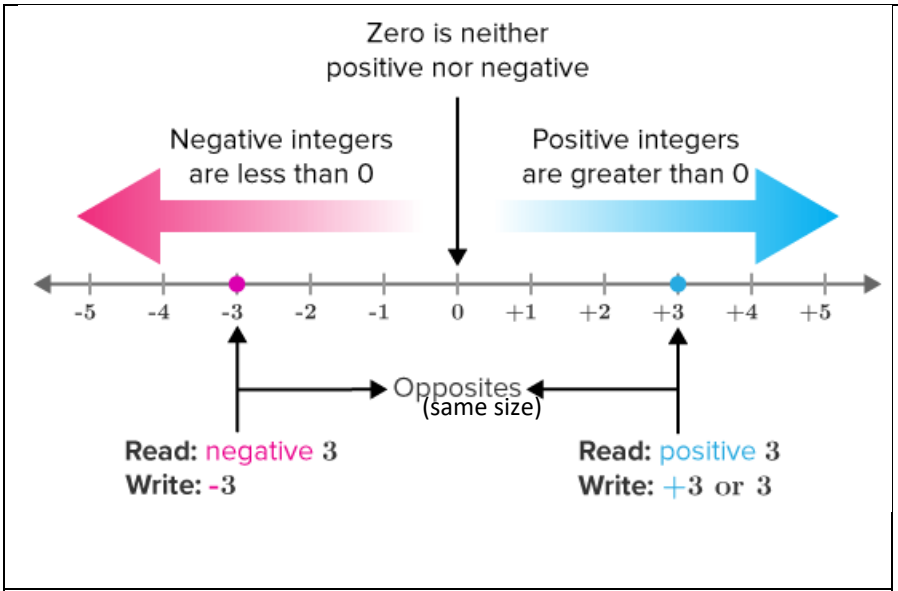
Component Knowledge

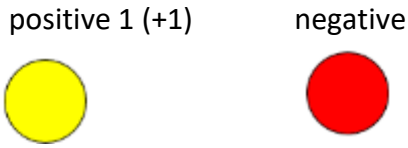
- Order positive and negative numbers
- Addition of positive and negative numbers
- Subtraction of positive and negative numbers
- Multiplication of positive and negative numbers
- Division of positive and negative numbers

Key Vocabulary

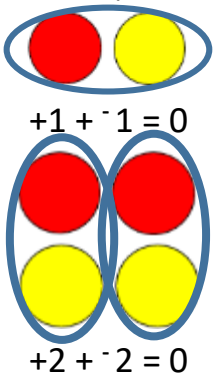
Integer	A whole number
Positive number	A number that has a value greater than zero
Negative number	A number that has a value less than zero
Zero pair	A set of two numbers that when added together equal zero.
Starting value	The starting value in a calculation
Ascending order	Numbers in order from smallest to largest
Descending order	Numbers in order from largest to smallest

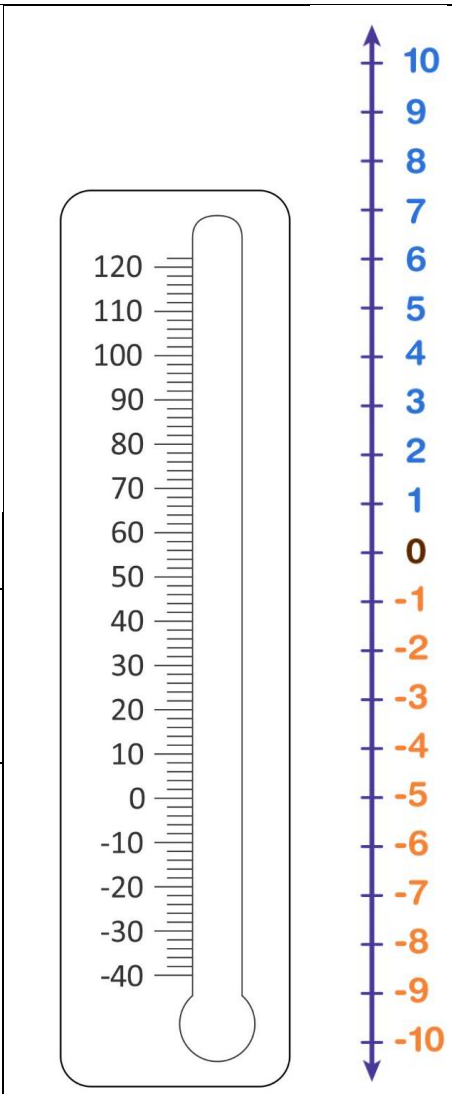
Representations of Directed Numbers





Zero pairs

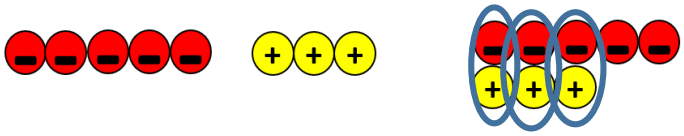




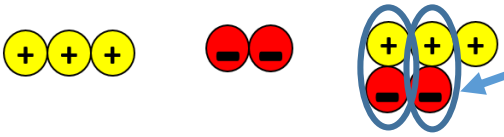
Adding with Directed Number

3 + 2 = 5

-5 + 3 = -2



3 + -2 = 1



-2 + -5 = -7

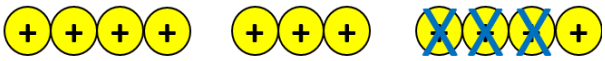
Adding positive numbers to the starting value increases the overall value.

Adding negative numbers to the starting value decreases the overall value. This has **the same effect as subtracting positive numbers**.

Subtracting with Directed Number

Sometimes we can directly subtract from the starting value.

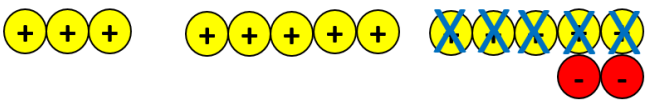
4 - 3 = 1



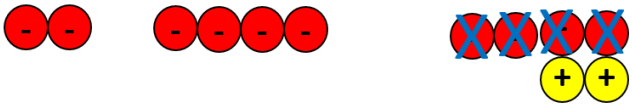
-4 - -3 = -1

Sometimes we **need to add zero pairs** to be able to subtract from the starting value.

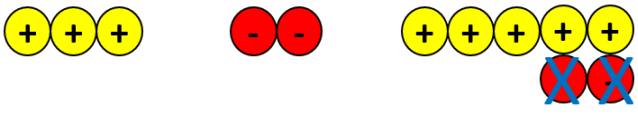
3 - 5 = -2



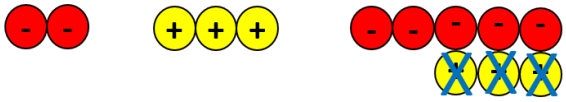
-2 - -4 = 2



3 - -2 = 5



-2 - 3 = -5



Subtracting positive numbers to the starting value decreases the overall value.

Subtracting negative numbers to the starting value increases the overall value.

Online clips

M527, M106

Four operations

Component Knowledge

- Multiply 2- and 3-digit numbers.
- Use bus stop method to divide numbers.
- Addition and subtraction.
- Know what the inverse of each of the four operations are.

Key Vocabulary

Multiplication	Adding a number to itself repeatedly.
Division	Splitting a number into equal parts.
Addition	The action or process of adding something to something else.
Subtraction	The process or skill of taking one amount from another.
Inverse	The opposite. The Reverse.

Multiplication Methods

Grid Method

14 x 26 = 364

x	10	4
20	200	80
6	60	24

200
80
60
24 +
364

Column method

625
x 26
3750
12500
16250

Add these two lines to get the final answer.

Remember to put a zero here as a place holder.

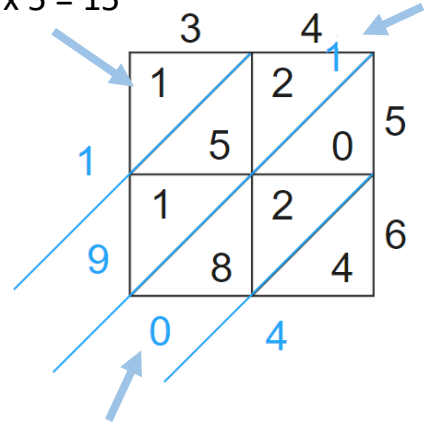
Chinese Method

34 x 56

3 x 5 = 15

8 + 2 + 0 = 10

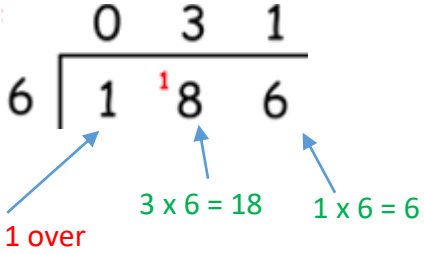
The one is carried.

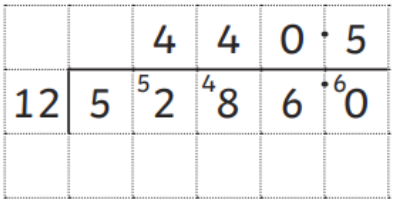


Add up along the diagonals and carry where necessary.

Division

186 ÷ 6 =





Addition/Subtraction

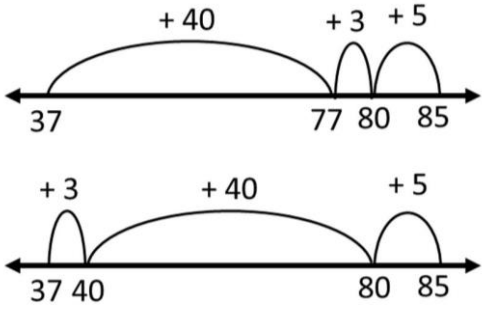
Column Method

$$\begin{array}{r} 5864 \\ +\ 3497 \\ \hline 9361 \\ \hline 1\,1\,1 \end{array}$$

- Starting with the ones, add each column in turn.
- Carry over tens, hundred and/ or thousands as required.

Number line

37 + 48



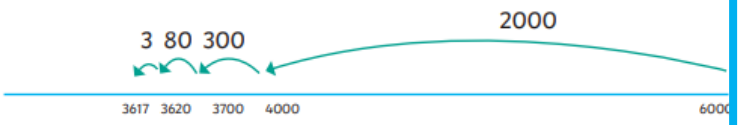
Column Method

Borrow tens, hundred and/ or thousands as required.

$$\begin{array}{r} 5\,\cancel{7}^{\,6}\,\cancel{4}^{\,13}\,\cancel{2}^{\,1} \\ -\ 3476 \\ \hline 2266 \end{array}$$

- Starting with the ones, subtract each column in turn.

Number line

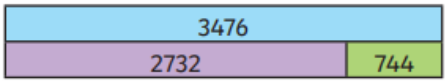


Calculate 6000 – 3617 = 2383

Inverse operation

Operation	Inverse
+	−
−	+
×	÷
÷	×

Using Inverse



3476 – 744 = 2732 can be checked using 2732 + 744 = 3476

This part whole shows the inverse calculations using these three numbers.

1549 + 2688 = 4237	2688 + 1549 = 4237
4237 – 1549 = 2688	4237 – 2688 = 1549

Online clips

M928, M347, M187, M354, M873, M262

Powers & roots

Component Knowledge

- Write repeated multiplication as a power
- Calculate small powers (powers of 2, 3 or 4) of any number by repeated multiplication
- Identify square and cube numbers
- Calculate square and cube roots

Key Vocabulary

Power	A notation and word used to show repeated multiplication of the same number
Index	Another term used for power
Square	*Squaring* a number means multiplying by itself (e.g. 2×2)
Square number	A *square number* is the result of squaring another number (e.g. 4 is a square number)
Cube	*Cubing* a number means multiplying by itself and itself again (e.g. $3 \times 3 \times 3$)
Cube number	A *cube number* is the result of cube another number (27 is a cube number – it's 3 cubed)
Square root	The number that when squared gives the answer wanted
Cube root	The number that when cubed gives the answer wanted

Notation

This is how powers are written.
It is a shorthand for repeated multiplication.
Other words used for power are **index** and **exponent**.
We read the power as '2 *to the power of* 4'
But there are also special terms for a power of 2 or 3. See below.

Index (Indices)

Power

Exponent

Base

$$2^4$$

$$2 \times 2 \times 2 \times 2$$

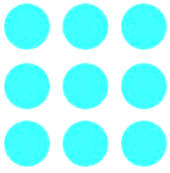
Squares

A square number is a number multiplied by itself.
As an example, 4 is a square number as 2 × 2 = 4
This can be written as 2^2 and read as "2 squared"
The ² symbol is how we write "squared"

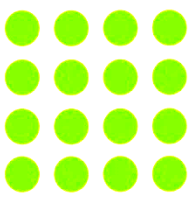
We call them "square numbers" because we can arrange them to make perfect squares.

$2 \times 2 = 4$

$3 \times 3 = 9$



$4 \times 4 = 16$



$5 \times 5 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

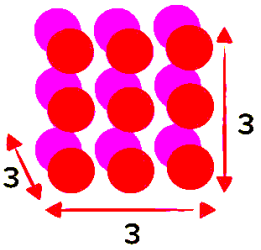
$11^2 = 121$

$12^2 = 144$

Cubes

A cube number is a number multiplied by itself 3 times. As an example, 8 is a cube number as 2 x 2 x 2 = 8
This can be written as 2^3 and read as "2 cubed".

We call them "cube numbers" because we can arrange them to make perfect cubes.



$3 \times 3 \times 3 = 27$

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

Roots of Squares and Cubes

The opposite of a square number is a square root.

Since 3^2 is 9, the square root of 9 is 3.

$9 = 3^2$

$\sqrt{9} = 3$

We use this symbol to represent the square root.

The opposite of a cube number is a cube root.

Since 2^3 is 8, the cube root of 8 is 2.

$8 = 2^3$

$\sqrt[3]{8} = 2$

We use this symbol to represent the cube root.

Other examples:

$\sqrt{25} = 5$ because $5^2 = 25$

$\sqrt{64} = 8$ because $8^2 = 64$

$\sqrt[3]{216} = 6$ because

$6 \times 6 \times 6 = 216$

This is read:

'the square root of 2'

'the cube root of 8'

Not all square roots are whole numbers. For example 110 is not a square number because $10^2 = 100$ and $11^2 = 121$.
To find $\sqrt{110}$ you would use a calculator:
$\sqrt{110} = 10.488 \ldots$

Online clip

M135

(BIDMAS) Order of operations

Component Knowledge

- Understand what order operations are completed and why that order is used.
- Understand how the order of operations can affect the answer.
- Apply order of operations to solve problems.

Key Vocabulary

Priority	The order of importance of something
Order	The rules that say which calculations comes first in an expression
Operation	A mathematical process
Brackets (Parentheses)	A pair of marks () that enclose figures or words
Index/Indices (Power)	The power (or exponent) of a number says how many times to use the number in a multiplication
Divide/Division	The process of separating somethings into parts
Multiply/ Multiplication	The process of grouping numbers
Addition	The process of adding two or more numbers together
Subtraction	The process of taking away one number from another

B	Brackets	$10 \times (4 + 2) = 10 \times 6 = 60$
I	Indices	$5 + 2^2 = 5 + 4 = 9$
D	Division	$10 + 6 \div 2 = 10 + 3 = 13$
M	Multiplication	$10 - 4 \times 2 = 10 - 8 = 2$
A	Addition	$10 \times 4 + 7 = 40 + 7 = 47$
S	Subtraction	$10 \div 2 - 3 = 5 - 3 = 2$

When performing calculations, you must follow the correct order of operations as shown in the table.

Note: BIDMAS may also be known as BODMAS or PEMDAS.

Brackets ⟶ (or) ⟵ Parenthesis
Order ⟶ √ or x^2 ⟵ Exponents

Division ⟶ ÷ or x ⟵ Multiplication
Multiplication ⟶ x or ÷ ⟵ Division

Addition ⟶ + or + ⟵ Addition
Subtraction ⟶ - or - ⟵ Subtraction

Basic examples involving operations, brackets and indices:

$a)\ 7 + 6 \times 3$
$= 7 + 18$
$= 25$

$b)\ 22 - 12 \div 4$
$= 22 - 3$
$= 19$

$c)\ (8 + 3) \times 9 \div 3$
$= 11 \times 9 \div 3$
$= 99 \div 3$
$= 33$

$d)\ 2 \times 3^2 \div 3$
$= 2 \times 9 \div 3$
$= 18 \div 3$
$= 6$

$e)\ (40 - 15) \div (15 \div 3)$
$= 25 \div 5$
$= 5$

Examples involving fractions:

a) $\frac{5+7\times4}{16-5}$
$= \frac{33}{11}$
$= 3$

b) $\frac{4\times12\div6}{14-6\times2}$
$= \frac{8}{2}$
$= 4$

Problem Solving:
Add brackets to make the following calculation correct

a) $18 - 3 + 4 \times 2 = 22$ $(18 - (3 + 4)) \times 2 = 22$

b) $18 - 3 + 4 \times 2 = 7$ $18 - (3 + 4 \times 2) = 7$

c) $3 \times 16 \div 4 \div 2 + 1 = 16$ $3 \times 16 \div (4 \div 2 + 1) = 16$

Online clips

M521

Factors, multiples & primes

Component Knowledge

- Identify factors and multiples
- Identify a prime number
- Complete a prime factor tree and write the number in index form
- Calculate HCF and LCM of 2 values using an appropriate method.

Key Vocabulary

Factor	Numbers that we can multiply together to get another number
Multiple	The result of multiplying a number by an integer
Prime	A number that only has two factors 1 and itself
Highest common factor	The greatest number that is a factor of 2 (or more) other numbers
Lowest common multiple	The smallest positive number that is a multiple of two or more numbers
Product	The answer when two or more values are multiplied together
Factorisation	Writing a number as a product of two or more smaller numbers
Integer	A whole number

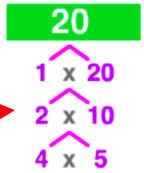
Multiples: The result of multiplying a number by an integer. It is the times table of a number.

Multiples of 4: 4, 8, 12, 16,20 …
Multiples of 5: 5, 10, 15, 20, 25….

Multiples are the list of times tables

Factors: A number that divides exactly into another number without a remainder. It is often helpful to write them in pairs.

Write them in pairs first so you don't miss any!



Factors of 20 = 1, 2, 4, 5, 10, 20

HCF & LCM

Highest common factor

Find the HCF of 12 and 20

Factors of 12	Factors of 20
1 and 12	1 and 20
2 and 6	2 and 10
3 and 4	4 and 5

4 is the highest factor of both numbers

Lowest common multiple

Find the lowest common multiple of 4 & 6

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32 …
Multiples of 6: 6, 12, 18, 24, 30, 36 …

12 is the lowest number that appears in both times tables.

Prime Numbers

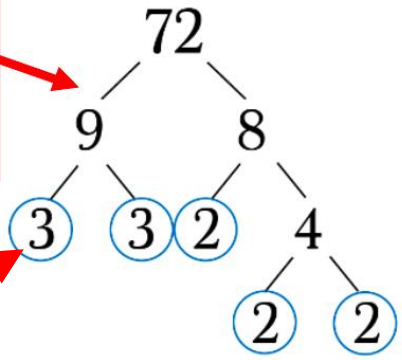
1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	49	50
51	52	**53**	54	55	56	57	58	**59**	60
61	62	63	64	65	66	**67**	68	69	70
71	72	**73**	74	75	76	77	78	**79**	80
81	82	**83**	84	85	86	87	88	**89**	90
91	92	93	94	95	96	**97**	98	99	100

Prime numbers

Prime Factorisation

Write 72 as a product of its prime factors

We need to find pairs of numbers that multiply to give the number above.



When you get a prime number circle it.

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$72 = 2^3 \times 3^2$$

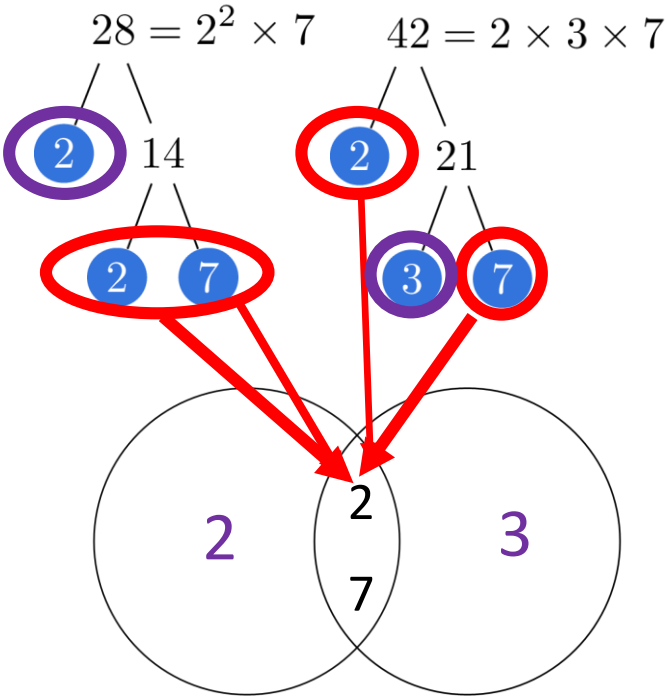
If a number is repeated we write it as a power

HCF & LCM using prime factors

Find the HCF and LCM of 28 and 42

First start by finding the Prime factors of 28 and 42

$28 = 2^2 \times 7$ $\quad$ $42 = 2 \times 3 \times 7$



Both trees have a 2 and 7 so those numbers go in the middle of the Venn diagram as they are shared.

The remaining numbers in the tree go in outside circles of the Venn

HCF – the highest common factor is found by multiplying the centre shared part of the Venn diagram

HCF = 2 x 7 = 14

LCM – the lowest common multiple is found by multiplying all the numbers in the Venn diagram

LCM = 2 x 2 x 3 x7 =84

Online clips

M823, M322, M108, M227, M698, M365

Measures

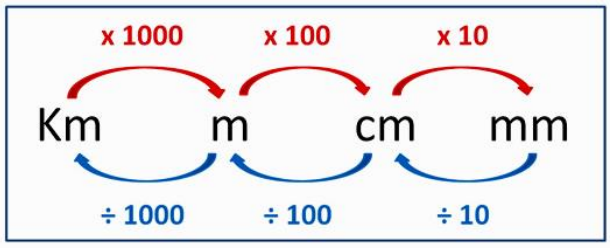
Component Knowledge

- Convert between units of length
- Convert between units of capacity
- Convert between units of mass
- Convert between units of time

Key Vocabulary

Convert	To change from one unit to another such as from centimetres to millimetres, or litres to millilitres, etc.
Unit	A quantity used as a standard of measurement
Length	The measurement of something from end to end
Capacity	The maximum amount that something can contain
Mass	The weight of an object
Time	*A numerical quantity that represents the duration between two events.*

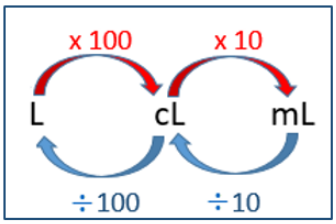
Units of length



5km = ? m Need to x 1000 5 x 1000 = 5000m ✓

120cm = ? m Need to ÷ 100 120 ÷ 100 = 1.2m ✓

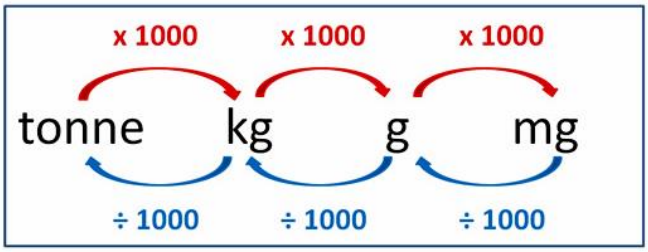
Units of capacity



5L = ? cL Need to × 100 5 × 100 = 500 cL

750 mL = ? cL Need to ÷ 10 750 ÷ 10 = 75 cL

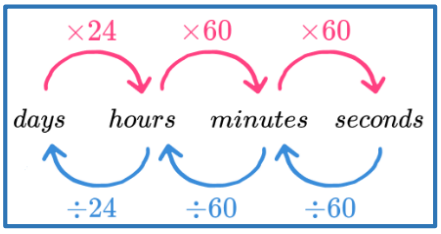
Units of mass



Mass conversions use 1000's, and usually create fairly large results.

1.6 tonne = ? kg Need to x 1000 1.6 x 1000 = 1600 kg ✓

Units of time



2 mins = ? secs need to × 60 2 × 60 = 120 secs

96 hrs = ? days need to ÷ 24 96 ÷ 24 = 4 days

Online clips

M772, M761, M530, M774, M627, M515

Properties of 2D shapes

Component Knowledge

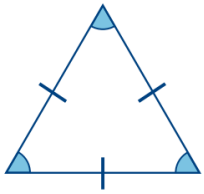
- Identify different types of triangles
- Describe the properties of different types of triangles
- Identify different quadrilaterals
- Describe the definitions and properties of quadrilaterals.

Key Vocabulary

Polygon	Is a flat two dimensional (2D) shape with straight edges that are all joined up.
Quadrilateral	Is a polygon that has four sides (edges), four angles and four corners (vertices).
Right angle	Is an angle of 90 degrees
Parallel	Two lines that stay the same distance apart for their entire length.
Perpendicular	A straight line is at 90° to another given line or surface
Line of Symmetry	A line that cuts a shape exactly in half. If you were to fold the shape in half both the sides would match exactly.

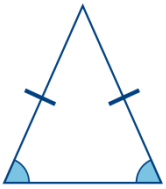
Properties of Triangles: There are 4 types of triangles;

Equilateral triangle:



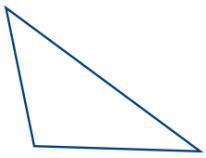
All sides are the same length.
All internal angles are the same (60°)

Isosceles triangle:



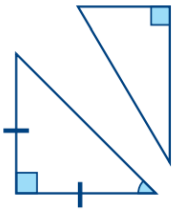
An isosceles triangle has two sides of equal length and two angles of equal sides.

Scalene triangle



A scalene triangle has no equal sides or angles.

Right-angled triangle

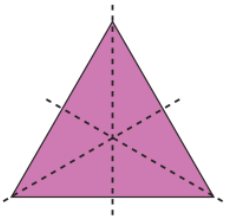


A right-angled triangle always has one 90° angle.
It can be isosceles or scalene

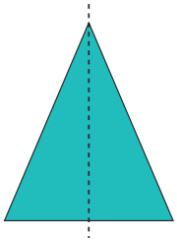
(Equal sides are shown on a diagram by a dash.)

Lines of symmetry may be horizontal, vertical or diagonal. Some 2D shapes will have no lines of symmetry and some 2D shapes will have multiple lines of symmetry.

An equilateral triangle has 3 lines of symmetry



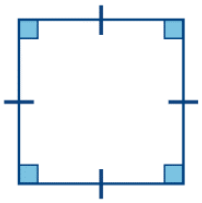
An isosceles triangle has one line of symmetry



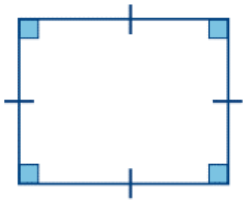
Properties of Quadrilaterals:

Equal sides are shown by a dash (/) , if there are two sets of equal sides the second is shown by two dashes (//)

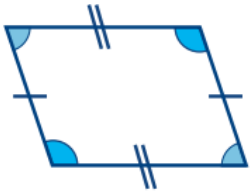
Parallel sides are shown by a set of arrows. (>)



A square has four sides of equal length and four right angles (90^{o}). It has two pairs of parallel sides. A square is also a special case of a rectangle, a rhombus and a parallelogram.

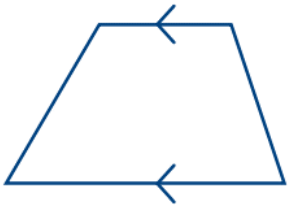


A rectangle as two pairs of parallel, equal sides and four right angles. A rectangle is also a parallelogram.

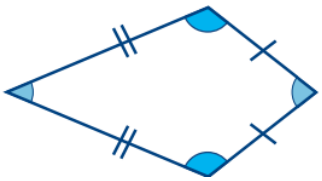


A parallelogram has two pairs of parallel, equal sides and opposite equal angles.

A rhombus has four sides of equal length and opposite equal angles. A rhombus is also a parallelogram.

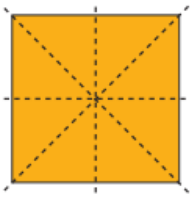


A trapezium only has one pair of opposite parallel sides.

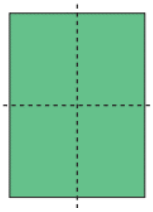


A kite has two pairs of adjacent equal sides and one pair of opposite equal angles.

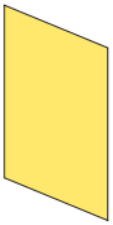
A square has four lines of symmetry



A rectangle has two lines of symmetry



A parallelogram has no lines of symmetry



Online clips

M814, M276, M523

Perimeter

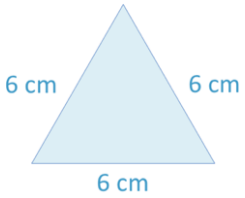
Component Knowledge

- Calculate the perimeter of a 2D shape.
- Calculate the length of a missing length of a side when given the perimeter of a 2D shape.
- Calculate the perimeter of a compound shape.

Key Vocabulary

Perimeter	The total distance around the outside of a shape.
Base	The bottom line of a shape
Height	The line from the bottom to the top of a shape
Compound shape	A shape made up of a combination of other known shapes put together.
2D shape	A two-dimensional (2D) shape can be defined as a flat figure or a shape that has two dimensions – base and height.

To calculate the perimeter of a shape, add the lengths of all the sides of the shape together.



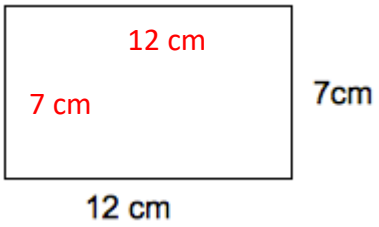
Perimeter

= 6cm + 6cm + 6cm

= 18 cm

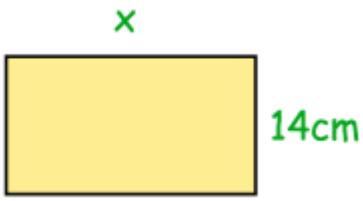
Remember the units need to be in the answer

When calculating the perimeter of a rectangle remember that parallel sides are equal.



Perimeter = 12 cm + 7 cm +12 cm +7 cm

Perimeter = 38 cm



Perimeter = 80cm

Form an equation:

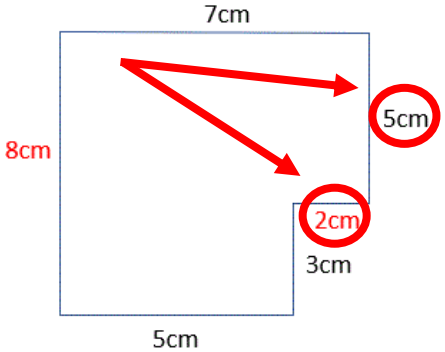
x + x + 14 + 14 = 80 cm

2x + 28 = 80 cm

2x = 52 cm

x = 26 cm

To calculate the perimter of a compoud shape you made need to calculate the length of missing sides.



Perimeter = 7cm +5cm + 2cm + 3cm + 5cm + 5cm + 8cm = 30cm

Online clips

M920, M635, M690

Area of 2-D shapes

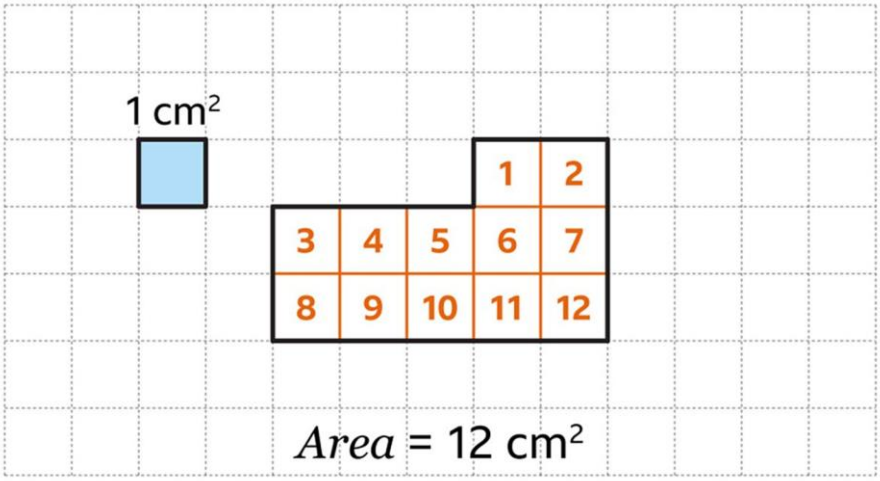
Component Knowledge

- Identify the relevant dimensions
- Identify the correct formula for area
- Use the correct formula to calculate the area of rectangles, triangles, parallelograms and trapeziums.
- Express the answer in the correct units

Key Vocabulary

Area	The amount of squared units that fit inside a shape
Dimension	The lengths of the sides of the shape
Unit of measure	This can be length (cm, mm, m) or area (cm^2, mm^2)
Compound shape	A 2-D shape composed of key 2-D shapes

Area is how much space fits inside a shape. We usually measure it in cm^2, this means how many 1cm squares can fit inside the shape.



Squares and rectangles:

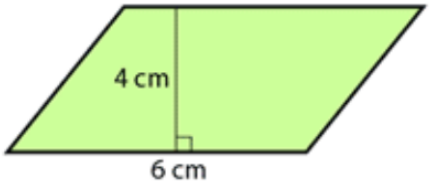
The formula is the same for both shapes: **A = Length x Width**

A = 3 x 3
= 9cm²

A = 10 x 4
= 40cm²

Parallelograms:

The formula is similar to a rectangle but instead of width we use the height. **A = Length x Height**



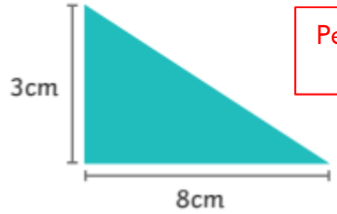
A = 6 x 4
= 24cm²

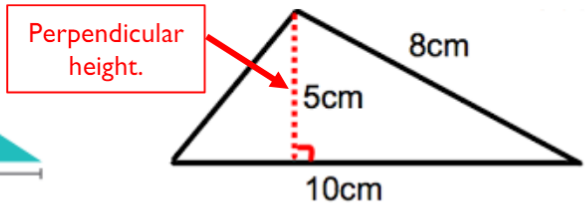
Sometimes the length is referred to as the base.

Triangles: To find the area of a triangle we use the following formula:

$$\textbf{Area} = \frac{\text{Base x perpendicular height}}{2}$$

The formula is very similar to a rectangle but we must divide by 2 because a triangle is half the size of a rectangle.





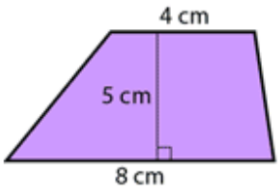
$$\text{Area} = \frac{8 \times 3}{2} = 12\text{cm}^2$$

$$\text{Area} = \frac{10 \times 5}{2} = 25\text{cm}^2$$

Trapeziums: To find the area of a trapezium we use the following formula:

$$\textbf{Area} = \frac{(a+b)}{2} \times h$$

Where a and b are the parallel sides and h is the height.



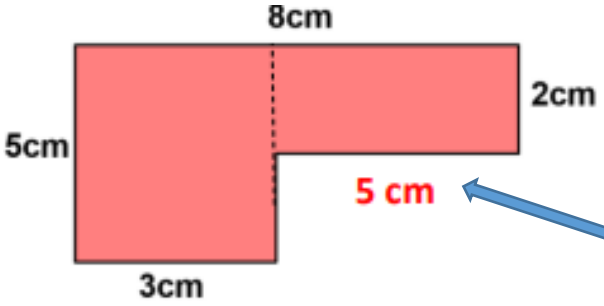
Area= 4 + 8 = 12 — Add the parallel sides.

12 ÷ 2 = 6 — Divide the total by 2.

6 x 5 = 30cm² — Multiply by the height.

Compound shape example

A compound shape is a shape made up of other shapes.



You must determine any missing dimensions,

e.g. 8 – 3 = 5cm

$$Area = (5 \times 3) + (2 \times 5)$$
$$= 25cm^2$$

Online clips

M900, M390, M291, M610, M269, M996