

Density, mass and volume



Component Knowledge

- Calculate simple density, mass or volume
- Calculate more complex density, mass or volume
- Combining mass and volume to find density of a compound.

Key Vocabulary

Density	A measure of how tightly the mass of an object is packed into the space it takes up. If an object is heavy and small it will have a higher density
Mass	The mass of an object is the quantity of matter it contains. It never changes.
Volume	Volume is defined as the space occupied within the boundaries of an object in three-dimensional space
Units	The unit of measure used to describe density, mass and volume.
Compound measurement	A measure made up of two or more measurements (e.g. speed, pressure, density)

Formulae for density, mass and volume

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$\text{Mass} = \text{Density} \times \text{Volume}$$

Calculate density

A solid silver spoon has a mass of 65.1g. The volume of the spoon is 6.2cm³. Calculate the density of silver.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \leftarrow \text{Write out the formula}$$

$$\text{Density} = \frac{65.1\text{g}}{6.2\text{cm}^3} \quad \leftarrow \text{Substitute in the values from the question}$$

$$\text{Density} = 10.5 \text{ g/cm}^3 \quad \leftarrow \text{Remember to include the units in the final answer}$$

Calculate volume

Iron has a density of 7.8g/cm³. A solid iron statue has a mass of 877.5g. Work out the volume of the statue.

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}} \quad \leftarrow \text{Write out the formula}$$

$$\text{Volume} = \frac{877.5\text{g}}{7.8 \text{ g/cm}^3} \quad \leftarrow \text{Substitute in the values from the question}$$

$$\text{Volume} = 112.5 \text{ cm}^3 \quad \leftarrow \text{Remember to include the units in the final answer}$$

Calculate mass

A piece of plastic has a density of 1.3g/cm³ and a volume of 100cm³. Work out the mass of the piece of plastic.

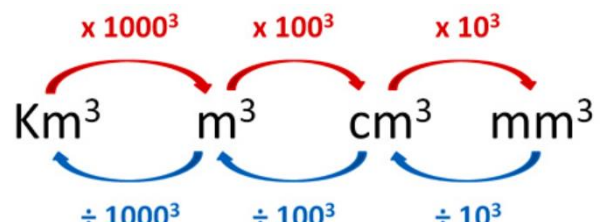
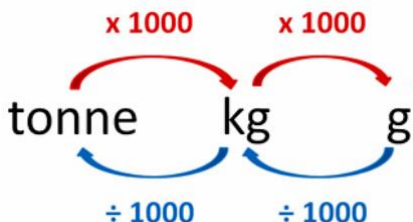
$$\text{Mass} = \text{Density} \times \text{Volume} \quad \leftarrow \text{Write out the formula}$$

$$\text{Mass} = 1.3\text{g/cm}^3 \times 100\text{cm}^3 \quad \leftarrow \text{Substitute in the values from the question}$$

$$\text{Mass} = 130\text{g} \quad \leftarrow \text{Remember to include the units in the final answer}$$

Useful

Conversions



Calculate more complex density, mass or volume

When calculating more complex density, mass or volume you may need to do a calculation before you can then substitute the values from the question into the formula. You may need to calculate the volume of the object first or you may need to change the units of mass or volume so that they are the same.

A glass cube of side length 5cm has a mass of 306.25g. Calculate the density of the glass.

$$5 \times 5 \times 5 = 125\text{cm}^3 \quad \leftarrow \text{Calculate the volume of the cube}$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \leftarrow \text{Write out the formula}$$

$$\text{Density} = \frac{306.25\text{g}}{125\text{cm}^3} \quad \leftarrow \text{Substitute in the values you know}$$

$$\text{Density} = 2.45\text{g/cm}^3 \quad \leftarrow \text{Remember to include the units in the final answer}$$

A garden ornament has a volume of 0.05m^3 .

The ornament is made from a stone that has a density of 6.4g/cm^3 . Calculate the mass of the ornament. Include suitable units.

$$0.05\text{m}^3 \times 1,000,000 = 50,000\text{cm}^3 \quad \leftarrow \text{Units need to be the same. Convert m}^3 \text{ into cm}^3$$

$$\text{Mass} = \text{Density} \times \text{Volume} \quad \leftarrow \text{Write out the formula}$$

$$\text{Mass} = 6.4\text{g/cm}^3 \times 50,000\text{cm}^3 \quad \leftarrow \text{Substitute in the values from the question}$$

$$\text{Mass} = 320,000\text{g} \quad \leftarrow \text{Remember to include the units in the final answer}$$

$$\text{Mass} = 320\text{kg} \quad \leftarrow \text{Change the units to kg as it is more suitable than g}$$

Combining mass and volume to find new density

When combining mass and volume to find a new combined density you cannot just add the two densities together. You have to find the total mass and the total volume of the new substance and then use these amounts to calculate the density of the compound (Sterling silver in the example below).

Some sterling silver is made with 900 g of silver and 90 g of copper. The density of silver is 10 g/cm^3 . The density of copper is 9 g/cm^3 . What is the density of the sterling silver?

a)

	Silver	Copper	Sterling silver
Density	10 g/cm^3	9 g/cm^3	
Mass	900g	90 g	
Volume			

\leftarrow Fill in what you know from the question

c)

	Silver	Copper	Sterling silver
Density	10 g/cm^3	9 g/cm^3	
Mass	900g	90 g	990 g
Volume	90cm^3	10 cm^3	100 cm^3

\leftarrow Calculate the new mass and new volume by adding

b)

	Silver	Copper	Sterling silver
Density	10 g/cm^3	9 g/cm^3	
Mass	900g	90 g	
Volume	90cm^3	10 cm^3	

\leftarrow Calculate the missing volumes

d)

	Silver	Copper	Sterling silver
Density	10 g/cm^3	9 g/cm^3	9.9 g/cm^3
Mass	900g	90 g	990 g
Volume	90cm^3	10 cm^3	100 cm^3

\leftarrow Calculate the new density by dividing the new mass by the new volume

Online clip

U910

Speed, Distance & Time



Component Knowledge

- Calculate speed given distance and time (including fractional time).
- Use the correct formula to calculate speed, distance & time.

Key Vocabulary

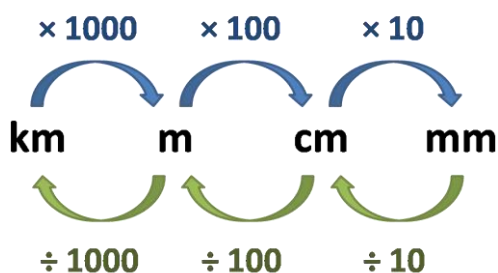
Speed	A measure of how fast something is happening
Distance	A measure of how far it is from one place to another
Time	A measure of how long something takes to happen
Units	A quantity used as a standard measurement
Convert	To change something from one form to another
Average	A calculated central value of a set of numbers
Metric	A standard unit of measure using metres, kilograms and seconds
Imperial	A unit of measure developed in England. E.g. miles, pounds, gallons etc

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

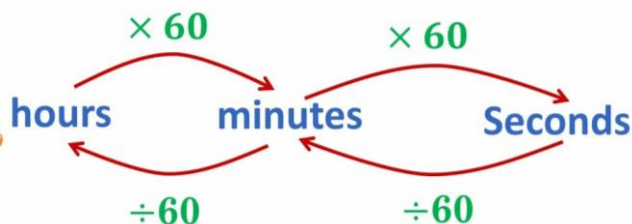
$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Useful conversions



5 miles \approx 8 kilometres



Units of speed include: m/s (metres per second), mph (miles per hour), Km/h (kilometres per hour).

Units of distance include: m (metres), km (kilometres) miles.

Units of time include: s (seconds), min (minutes), h (hours).

Example 1

Jim travels 45 miles in 3 hours.

What was his average speed in mph?

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{45}{3}$$

$$= \underline{15\text{mph}}$$

Example 2

Jess travels 45 miles in 1 hour 30 mins.

What was her average speed in mph?

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{45}{1.5}$$

$$= \underline{30\text{mph}}$$

1 hour 30 mins
= $1 \frac{30}{60}$ h
= 1.5 hours

Example 3

Jim drives at 40 mph for 3 hours.

How far did he travel?

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\begin{aligned} \text{Distance} &= 40 \times 3 \\ &= \underline{120 \text{ miles}} \end{aligned}$$

Example 4

For 15 minutes Sally ran at an average speed of 20 km/h

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\begin{aligned} \text{Distance} &= 20 \times 0.25 \\ &= \underline{5 \text{ km}} \end{aligned}$$

Note: there are different units of time so we convert mins to hours.

$$15 \text{ mins} = \frac{15}{60} = 0.25 \text{ h}$$

Example 5

A train travels 300 miles at 60 mph.

How long did this take?

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{300}{60}$$

$$= \underline{5 \text{ hours}}$$

Example 5

A runner travels 3 km at 5 m/s.

How long did they take?

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{3000}{5}$$

$$= \underline{600 \text{ seconds}}$$

$$= \underline{10 \text{ minutes}}$$

Note: there are different units of distance so we convert km to m.

$$3 \text{ km} = 3000 \text{ m}$$

Note: this is not a sensible unit. We convert 600s to mins.

$$600 \text{ s} = 10 \text{ mins}$$

Multi-Part Journeys

Julie drove 45km from Bath to Bristol.

She then drove 68km from Bristol to Cardiff.

Julie's average speed from Bath to Bristol was 50km/h

Julie took 105 minutes to drive from Bristol to Cardiff.

Work out Julie's average speed for her total drive from Bath to Cardiff.

Creating a table can help solve problems with multi-part journeys.

We cannot just find the second speed and take the mean of the 2 values because the distances are different.

	Speed	Distance	Time
Bath to Bristol	50 km/h	45 km	0.9 h
Bristol to Cardiff	We do not need this	68 km	105 mins (1.75 h)
Total	42.6 km/h	113 km	2.65 h

Use the formula to find the missing value so we can find the total distance and total time by adding them (we must convert the time to a decimal).

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{113}{2.65} = 42.6 \text{ km/h (1. d. p)}$$

Online clips

U151, M515

Real life graphs



Component Knowledge

- Plot and interpret simple real life graphs
- Plot and interpret distance time graphs

Key Vocabulary

Real life graph	This is a graph that represents a situation that we would see in real life.
Distance time graph	A graph that shows a journey and the relationship between the distance reached in a given time.
Y-intercept	Where a graph crosses the y-axis.
Gradient	How steep a line is at any point
Gradient (distance time graph)	The rate of change of one variable with respect to another (distance and time). This can be seen by the steepness and represents speed.

Real life graphs

Graphs that are representative of real-life situations. The actual meaning of the values depends on the labels and units on each axis.

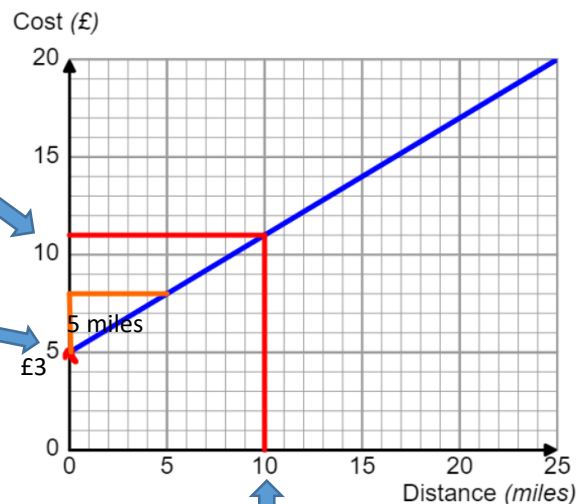
Real life graph – taxi journey

This graph shows the cost of using a taxi for a journey.

The gradient shows the cost per 5 miles travelled.
In this example it costs £3 per 5 miles travelled, which equals £0.60 per miles travelled.

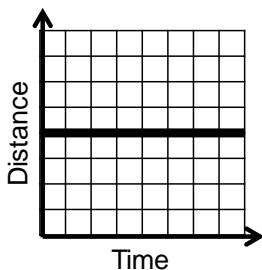
The y-intercept shows the starting cost for the journey (something that has to be paid no matter how long the journey is). In this example the starting cost is £5.

The graph can be used to calculate the cost of a journey or the distance of the journey.



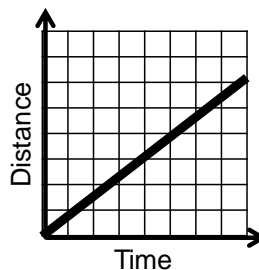
Using this graph, a journey of 10 miles costs £11.

Introduction to distance time graphs



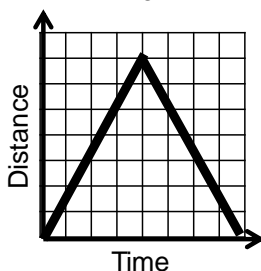
A horizontal line means that there is no movement.

Example - a car **remains parked** in a car park.



A diagonal line means that there is movement at a constant rate. The less steep the gradient is, the slower the movement is.

Example - a motorbike travels away from home at a **steady speed**.



A diagonal line means that there is movement at a constant rate. If it is positive (up) it means it is movement away from the start. If it is negative (down) it means it is movement back to the start.

Example - a runner runs at a **steady pace** to the end of a track, turns around then runs at the **same speed back**.

Real life graphs - distance time graphs

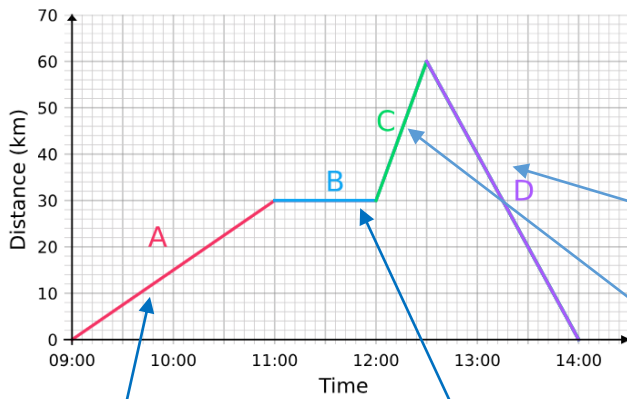
The graph below describes a journey that has several parts to it, each represented by a different straight line.

Part A: 09:00–11:00, the person travelled 30 km away from their starting point and that took them 2 hours.

Part B: 11:00–12:00, we can see that the line is flat, so the distance from their starting point did not change – they were stationary.

Part C: 12:00–12:30, they moved a further 30 km away from their starting point.

Part D: 12:30–14:00, they travelled the full 60 km back to where they began.



Calculate the speed – for each part of the journey:

$$\text{Speed}(S) = \frac{\text{Distance}(D)}{\text{Time}(T)}$$

Part D: 12:30–14:00

$$\text{Speed}(S) = \frac{60}{1.5} = 40 \text{ km/h}$$

Part A: 09:00–11:00

$$\text{Speed}(S) = \frac{30}{2} = 15 \text{ km/h}$$

Part B: 11:00–12:00

$$\text{Speed}(S) = \frac{0}{1} = 0 \text{ km/h}$$

Part C: 12:00–12:30

$$\text{Speed}(S) = \frac{30}{0.5} = 60 \text{ km/h}$$

From this we can see that the person travelled the fastest over **part C**.

Online clips

U652, U638, U896, U403, U914

Area under a graph



Component Knowledge

- Know that distance comes from finding the area under the graph
- Find the total area under the graph using trapezia, triangles and rectangles.

Key Vocabulary

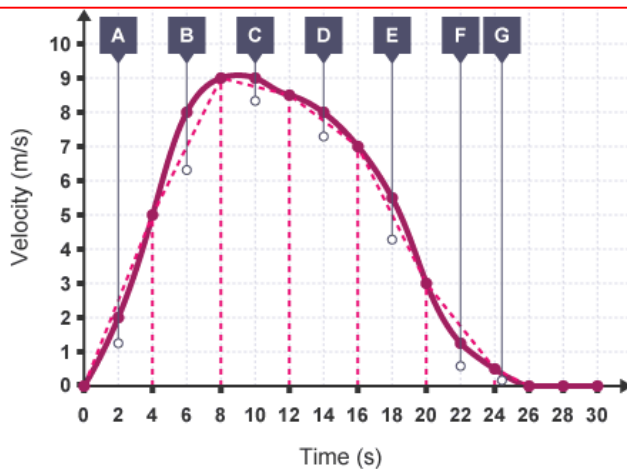
Distance	The length of the space between two points
Speed	The rate at which something moves or operates
Time	The measurable period during with an action continues
Area	The measurement of a surface
Approximate	Something that is similar but not exactly equal to something else
Speed-time graph	A graph that shows the relationship between the speed of an object and the time elapsed

The area under a speed-time graph represents the distance travelled. The area under a velocity-time graph represents the displacement of the moving object. If the velocity is always positive, then the displacement will be the same as the distance.

The area under a graph can be estimated by dividing the space into triangles, rectangles and trapezia. The more shapes used, the more accurate your answer.

Example

The velocity of a sledge as it slides down a hill is shown in the graph.
Find the distance travelled by the sledge over its 30 seconds journey.
Vertical lines every 4 seconds along the horizontal axis have been added and points joined to make triangles, rectangles or trapeziums.



The areas of the shapes are

$$A: A = \frac{b \times h}{2} = \frac{4 \times 5}{2} = 10$$

$$B: A = \frac{(a+b) \times h}{2} = \frac{(5+9) \times 4}{2} = 28$$

$$C: A = \frac{(a+b) \times h}{2} = \frac{(9+8.5) \times 4}{2} = 35$$

$$D: A = \frac{(a+b) \times h}{2} = \frac{(8.5+7) \times 4}{2} = 31$$

$$E: A = \frac{(a+b) \times h}{2} = \frac{(7+3) \times 4}{2} = 20$$

$$F: A = \frac{(a+b) \times h}{2} = \frac{(3+0.5) \times 4}{2} = 7$$

$$G: A = \frac{b \times h}{2} = \frac{0.5 \times 2}{2} = 0.5$$

Total area = 131.5.

So, the total distance covered is 131.5m

Online clips

U265, U882

Gradient of a graph



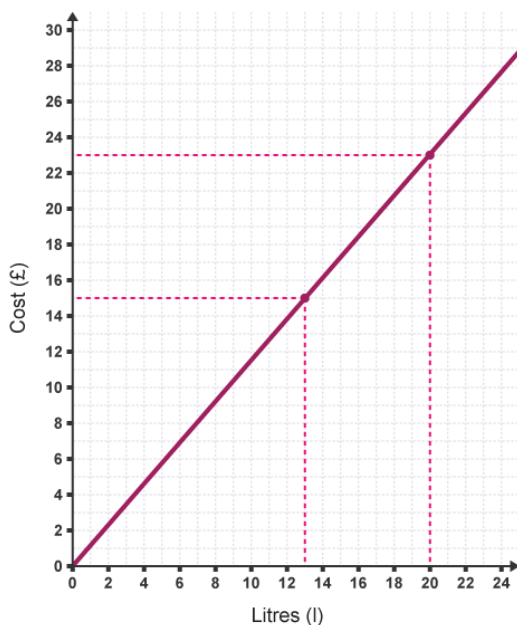
Component Knowledge

- Interpret what the gradient of a graph represents in real life
- Find the gradient of a straight line graph
- Find the gradient of a tangent to a curve

Key Vocabulary

Gradient	How steep a line is.
Tangent	A line that touches a curve at a point, matching the curve's slope at that point.
Curve	A smooth flowing line (no sharp changes).
Co-ordinate	A point on a graph showing how far along and how far up or down the point is from the origin.
Acceleration	How fast velocity changes.
Speed	How fast something is moving.

This graph shows the cost of petrol. It shows that 20 litres will cost £23 or £15 will buy 13 litres

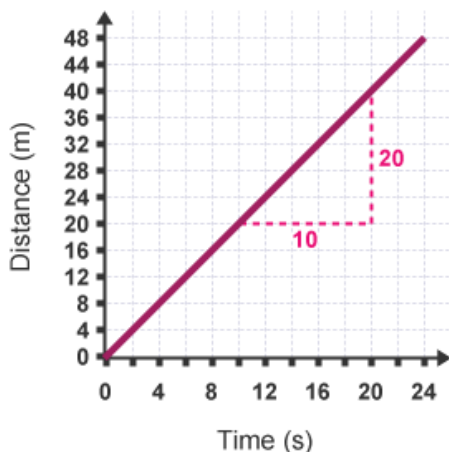


$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

Using the points (0,0) and (20,23) the gradient = $\frac{23-0}{20-0} = 1.15$

The units of the axes help give the gradient a meaning

In this example, the gradient shows the cost per litre (£1.15 per litre)

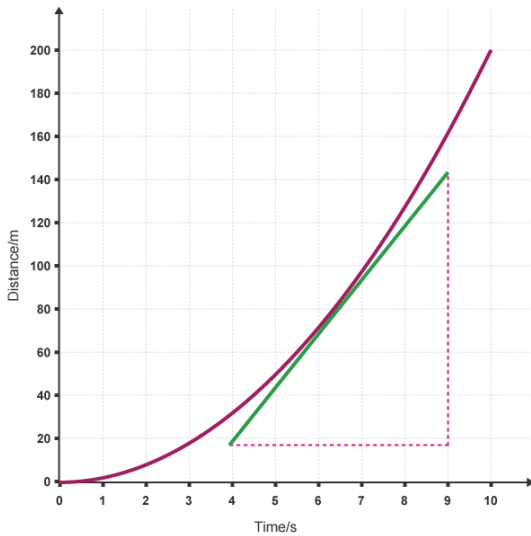


The gradient of a distance-time graph represents speed

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{change in metres}}{\text{change in seconds}} = m/s$$

The speed is $\frac{20}{10} = 2m/s$

This distance-time graph shows the first ten seconds of motion for a car.



The average speed over the 10 seconds = the gradient of the line from (0,0) to (10, 200) = $\frac{200}{10} = 20m/s$

To find an estimate of the speed after 6.5 seconds, draw the tangent to the curve at 6.5

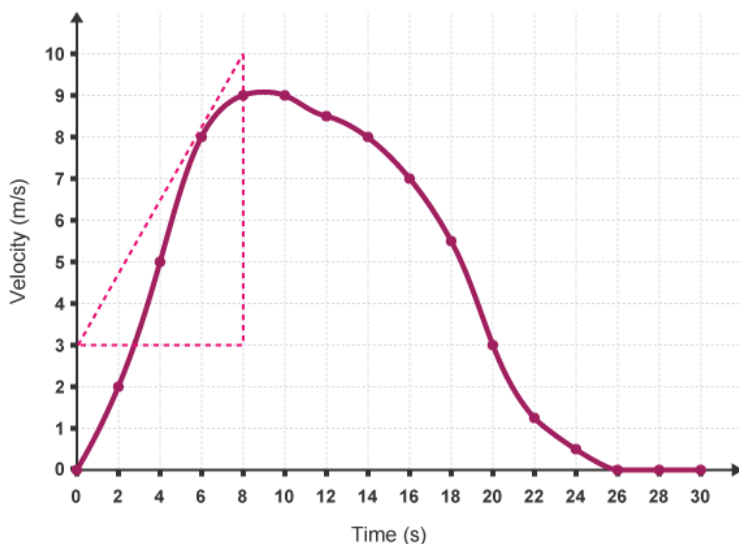
$$\text{Gradient} = \frac{140-20}{9-4} = \frac{120}{5} = 24m/s$$

A velocity-time graph shows the velocity of a moving object on the vertical axis and time on the horizontal axis.

The gradient of a velocity-time graph represents acceleration, which is the rate of change of velocity. If the velocity-time graph is curved, the acceleration, at a particular point in time, can be found by calculating the gradient of a tangent to the curve.

A negative gradient shows the rate of "slowing down" or deceleration.

The velocity of a sledge as it slides down a hill is shown in the graph. Find the acceleration of the sledge when $t = 6s$



Draw a tangent to the curve at the point where $t = 6s$ and draw two lines to form a right angle triangle. The acceleration is equal to the gradient of the tangent.

$$\begin{aligned} \text{Acceleration} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{7 \text{ m/s}}{8s} = 0.875m/s^2 \end{aligned}$$

After about 10 seconds, the gradients are negative meaning the sledge is slowing down or decelerating.

Online clips

U315, U477, U800

Probability

Rules



Component Knowledge

- Understand what probability shows
- Understand probability notation
- Write a probability of a single event
- Use the NOT rule
- Use the OR rule
- Use the AND rule

Key Vocabulary

Probability	The mathematical chance, likelihood, of an outcome happening
Event	The "thing" that is being completed/done/observed/counted
(Event) Outcome	What happens when the event is performed
Probability scale	A numerical scale from 0 to 1, with 0 being an impossible outcome and 1 being an outcome certain to happen
Mutually exclusive (event) outcomes	When outcomes cannot happen at the same time e.g. being an adult and being a child, you cannot be both
Exhaustive (event) outcomes	When a set of outcomes cover all possibilities with no gaps e.g. You pass a test or fail a test.
Independent events	Where the outcome of one event does not affect the outcome of another
Dependent events	Where the outcome of one event does affect the outcome of another

Single Event Probability:

The probability of an (event) outcome A , happening is

$$P(\text{outcome } A) = \frac{\text{number of ways outcome } A \text{ can happen}}{\text{number of ways any outcome can happen}}$$

e.g. the probability of rolling a number greater than 4 on a regular 6 sided dice

Outcomes "greater than 4": 5 or 6, so 2 options

All possible outcomes: 1, 2, 3, 4, 5 or 6, so 6 possibilities altogether

$$P(\text{roll a number greater than 4}) = \frac{2}{6}$$

Probability NOT happening:

The probability of an (event) outcome A , not happening is written as A' and is found by

$$P(A') = 1 - P(A \text{ does happen})$$

This is because the probabilities of mutually exclusive and exhaustive events always sum to 1.

Probability of A **OR** B happening:

The probability of either (event) outcome A happening, OR either (event) outcome B happening is written as $A \cup B$

$$P(A \cup B) = P(A) + P(B)$$

Eg If the probability I draw a tennis match is 0.4, $P(\text{draw})=0.4$
and the probability I win a tennis match is 0.3, $P(\text{win})=0.3$

The probability I either win OR draw is

$$\begin{aligned}P(\text{Win or Draw}) &= P(\text{Win}) + P(\text{Draw}) \\ &= 0.3 + 0.4 \\ &= 0.7\end{aligned}$$

Probability of A **AND** B happening:

For independent events the probability of (event) outcome A happening, AND then (event) outcome B happening is written as $A \cap B$

$$P(A \cap B) = P(A) \times P(B)$$

Eg If the probability I miss the bus is 0.3, $P(\text{miss the bus})=0.3$
and the probability I pass a test is 0.8, $P(\text{pass test})=0.8$

The probability I miss the bus and pass a test is

$$\begin{aligned}P(\text{Miss and Pass}) &= P(\text{Miss}) \times P(\text{Pass}) \\ &= 0.3 \times 0.8 \\ &= 0.24\end{aligned}$$

Online clip

M755

Venn Diagrams



Component Knowledge

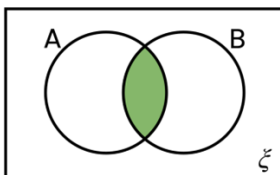
- Complete a Venn Diagram when given a set of data
- Fill in missing values in a Venn Diagram
- Interpret a Venn diagram
- Find probabilities from a Venn Diagram
- Use simple set notation

Key Vocabulary

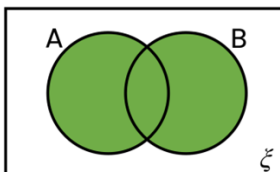
Set	A collection of "things" (objects or numbers)
Union	The set made by combining the elements of two sets
Intersection	The intersection of two sets has only elements common to both sets
Probability	The change that something happens
Venn Diagram	A diagram that shows sets which elements belong to which set by drawing regions around them. It is used to represent data that has an overlap.

Key Concepts

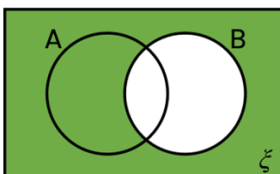
Venn diagrams show all possible relationships between different sets of data.



$A \cap B$
The **intersect** of A and B.
The set of elements in **both A and B**.



$A \cup B$
The **union** of A and B.
The set of elements in **A or B or both**.



B'
The **complement** of B.
The set of elements **not in B**.

Example

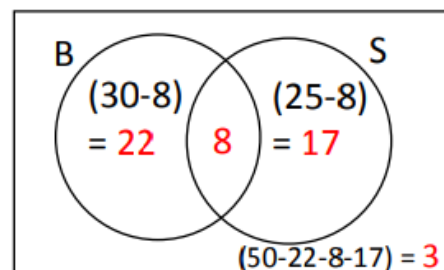
Out of 50 people surveyed:

30 have a brother

25 have a sister

8 have both a brother and a sister

This is what the Venn Diagram for this information would look like



Remember – the people in the intersection are also included in the whole circle so we don't duplicate data.

From the Venn Diagram, we can see that the probability of someone from this group just having a brother is $\frac{22}{50}$.

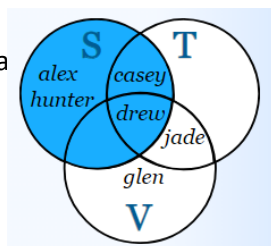
The probability of someone from this group having neither a brother or a sister is $\frac{3}{50}$.

The probability of having a brother and a sister,

$$P(A \cap B) = \frac{8}{50}$$

Venn Diagrams with 3 sets

Diagrams can be drawn to show more than 2 sets of data
This is an example of a Venn Diagram containing 3 sets.



$S = \{\text{Alex, Hunter, Casey and Drew}\}$

$T = \{\text{Jade, Casey and Drew}\}$

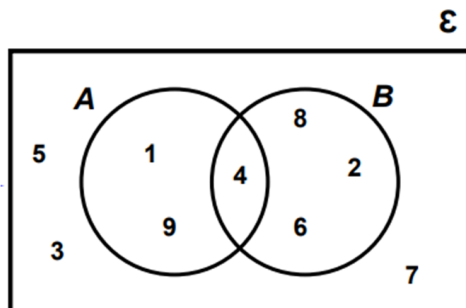
$V = \{\text{Drew, Jade and Glen}\}$

Example: Given a set of numbers

$\mathcal{E}=\{1,2,3,4,5,6,7,8,9\}$

$A=\{\text{square numbers}\}$

$B=\{\text{even numbers}\}$



\mathcal{E} - denotes the universal set.
This is the set containing all of the
elements being considered.

In set A 'the square numbers' are 1, 4 and 9.

In set B the 'even numbers' are 2, 4, 6, 8.

4 is in both groups so would go in the centre (the intersection)

Outside of the circles are any numbers remaining in \mathcal{E}

Online clips

M829, M419, M834

Set

Notation



Component Knowledge

- Complete a Venn Diagram when given a set of data
- Fill in missing values in a Venn Diagram
- Find probabilities from a Venn Diagram

Key Vocabulary

Set	A collection of "things" (objects or numbers)
Union	The set made by combining the elements of two sets
Intersection	The intersection of two sets has only elements common to both sets
Complement	All elements from a universal set not in our set
Element	Things contained in a set

Key Concepts

A set can be a list of items known as elements

A subset would be a selection of these elements.

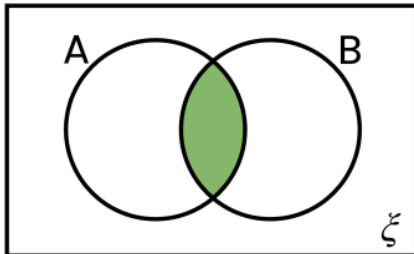
When we list elements within a set, we use these curly brackets { } and separate each elements in the list with commas.

The universal set, ξ , is the list of every element that there is available to choose from.

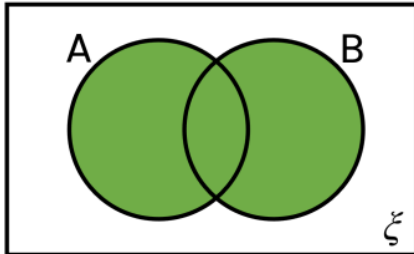
The complement of a set is denoted with an apostrophe and would be the remaining elements in the universal set that are not part of that set.

Symbol	Description
{ }	Curly brackets - contain all items in a set
,	Comma - separates items in a set
'	Complement - the items not in a set
ξ	The Universal Set - contains all items in every set and subset required
ϕ	The Empty Set - contains no items
A	Set A
A'	Not Set A (the complement of Set A)
B	Set B
B'	Not Set B (the complement of Set B)
$A \cap B$	A and B (A intersection B)
$(A \cap B)'$	Not A and B (the complement of A intersection B)
$A \cup B$	A or B (A union B)
$(A \cup B)'$	Not A or B (the complement of A union B)
$n(A)$	The number of elements in A. The cardinality of A

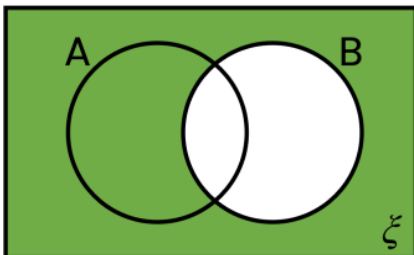
These are the different symbols you may see when working with set notation



$A \cap B$
 The **intersect** of A and B.
 The set of elements in **both A and B.**



$A \cup B$
 The **union** of A and B.
 The set of elements in **A or B or both.**



B'
 The **complement** of B.
 The set of elements **not in B.**

The shaded sections of the Venn Diagrams show which elements would be included for an intersection, a union or a complement

Example

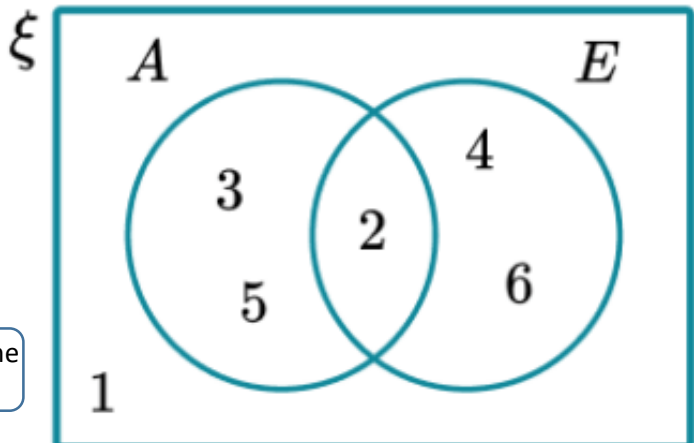
$\xi = \{1, 2, 3, 4, 5, 6\}$

The universal set shows us all the elements in the set

$A = \{2, 3, 5\}$

$E = \{2, 4, 5\}$

Sets A and E are subsets of the universal set



The **complement** of A (not A) is **$A' = \{1, 4, 6\}$**

The **union** of A and E (A or E) is **$A \cup E = \{2, 3, 4, 5, 6\}$**

The **intersection** of A and E (A and E) is **$A \cap E = \{2\}$**

Online clips

U748

Averages



Component Knowledge

- To understand and calculate the mode from a list.
- To understand and calculate the median from a list.
- To understand and calculate the mean from a list
- To calculate the range and understand it is **not** an average.

Key Vocabulary

Data set	Collection of values that share a common relationship. This could be answers to a set question or information for a set objective.
Average	Is a value (or values) that is used to represent a whole data set
Mode	The most frequent value in a data set. It is a type of average. Modal is another word used more mode.
Median	The middle value of a data set, when ordered. It is a type of average.
Mean	A measure of the size of the data when shared out equally. It is a type of average.
Range	A value to show spread out a data set is. It can be used to describe how representative of the whole data set the average used is. IT IS NOT AN AVERAGE.

Averages

We use averages to summarise a whole data set in a single value/few values. We do this so we can interpret large data sets and also compare data sets more easily.

Mode- the most frequent value/ few values in a data set. There can also be no mode in a set of data.

Ex 1, find the mode:

blue red blue green blue blue
pink green blue red blue yellow Blue is the mode.

Ex 2, find the mode:

9, 4, 3, 6, 9, 5, 2, 1, 8, 7

To make it easier, we can re-write these values in ascending(increasing) order.

1, 2, 3, 4, 5, 6, 7, 8, 9, 9. We can now see clearly 9 is the mode.

Ex 3, find the mode:

9, 4, 3, 6, 9, 5, 2, 1, 8, 7, 3

Re-written 1, 2, 3, 3, 4, 5, 6, 7, 8, 9, 9 We can see 3 and 9 are the modal values.

**** We usually only have 1, 2 or 3 modal values****

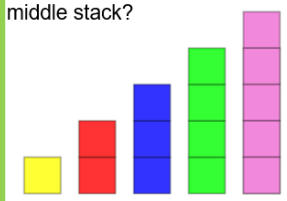
Ex 4, find the mode:

4, 3, 6, 9, 5, 2, 1, 8, 7

Re-written 1, 2, 3, 4, 5, 6, 7, 8, 9 We can see there are NO modal values.

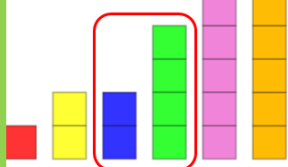
Median- the middle value in a data set, when in order. If there are 2 middle values, we find the midpoint between them.

How many blocks are in the middle stack?

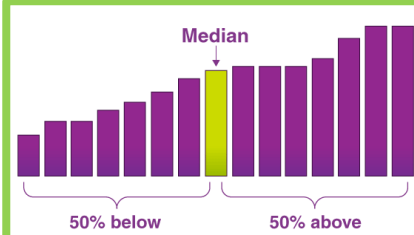


The middle stack has 3 blocks in

How many blocks are in the middle stack?



There is no "middle stack". We have to calculate the middle of 2 and 4. The middle would be 3.



Find the median of: ~~1~~, ~~3~~, ~~3~~, ~~6~~, ~~7~~, ~~8~~, ~~9~~

Median = 6

Find the median of: ~~1~~, ~~2~~, ~~3~~, ~~4~~, ~~5~~, ~~6~~, ~~8~~, ~~9~~

Median is the midpoint of 4 and 5 = 4.5

Find the median of the following set of numbers.

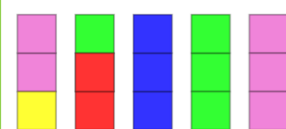
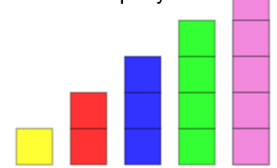
40 -2 10 40 -31 3 -34 -13 -10 1 30 16 -16

-34 -31 -16 -13 -10 -2 1 3 10 16 30 40 40



Mean- The mean is the size of each part when a quantity is shared equally. We can do this by adding all the values in the data set together and then dividing it equally between the number of values.

How many blocks would there be in each stack if they were shared out equally?



There would be three in each pile so the mean = 3

Example 1.
Find the mean of the following set of numbers.

19, 6, 17, 6

Solution.
To find the mean divide the sum of the numbers by the number of numbers.

$$\begin{aligned} \frac{\text{Sum of numbers}}{\text{Number of numbers}} &= \frac{19 + 6 + 17 + 6}{4} \\ &= \frac{48}{4} \\ &= 12 \end{aligned}$$

There are 4 values in the data set so we are dividing by 4.

Range- the range shows how spread out the data is. It is useful to order the data when finding the range. The smaller the range, the more consistent the data.

E.g. Find the range of the following numbers

43 36 10 -8 -3 -6 -4 -22

-22 -8 -6 -4 -3 10 36 43



Range = $43 - (-22) = 65$

Online Clips

M841, M934,

M940, M328

Averages from a frequency table



Component Knowledge

- To be able to calculate the mean, median, mode and range from a frequency table.

Key Vocabulary

Frequency	The number of pieces of data we have.
Mean	Add up the values you are given and divide by the number of values you have.
Median	The middle value when the data is in order.
Mode	The value or item with the highest frequency.
Range	This is the difference between the largest and smallest values. Shows the spread of the data

A team played 10 games and recorded the number of goals scored in those games.

Goal scored (x)	Frequency (f)	Total Frequency so far	(fx) (f multiplied by x)
0	2	(2) 2	$0 \times 2 = 0$
1	2	(2+2) 4	$1 \times 2 = 2$
2	5	(2+2+5) 9	$2 \times 5 = 10$
3	1	(2+2+5+1) 10	$3 \times 1 = 3$
Total	10		15

Calculating the mean number of goals scored.

Step 1: calculate the total frequency

Step 2: calculate (fx)

Step 3: calculate the mean using the formula $\frac{\text{total } fx}{\text{total frequency}}$

$$\text{Mean} = \frac{15}{10} = \underline{1.5 \text{ goals}}$$

Calculating the mode number of goals scored.

Mode = highest frequency of goals scored

Highest frequency = 5 for 2 goals scored

Mode = 2 goals scored

Calculating the median number of goals scored.

$$\text{Median value} = \frac{\text{Total frequency} + 1}{2}$$

$$\frac{11}{2} = 5.5^{\text{th}} \text{ value}$$

Median = 2 goals

add the frequency column until you reach the value in-between the 5th and 6th value

Calculating the range number of goals scored.

Highest number of goals = 3

Lowest number of goals = 0

Range = 3 - 0

Range = 3

Averages from a grouped frequency table



Component Knowledge

- Calculate an estimate for the mean from a grouped frequency table.
- Calculate the modal class interval from a grouped frequency table.
- Calculate the median from a grouped frequency table.

Key Vocabulary

Average	A number expressing the central or typical value in a set of data, particularly the mode, median or mean.
Grouped Data	If we have a large spread of data, we put it into categories (classes) to make the data easier to display or analyse.
Class interval	Group.

Averages from grouped data

a) Find an estimate for the mean of this data.

Length (L cm)	Frequency (f)	Midpoint (x)	fx
$0 < L \leq 10$	10	5	$10 \times 5 = 50$
$10 < L \leq 20$	15	15	$15 \times 15 = 225$
$20 < L \leq 30$	23	25	$23 \times 25 = 575$
$30 < L \leq 40$	7	35	$7 \times 35 = 245$
Total	55		1095

Step 1: Calculate the total frequency.

Step 2: Find the midpoint of each group.

Step 3: frequency (f) x midpoint (x).

Step 4: Calculate the estimated mean.

$$\frac{\text{Total } fx}{\text{Total } f} = \frac{1095}{55} = 19.9\text{cm}$$

b) Identify the modal class interval.

Modal class is $20 < L \leq 30$

Modal Class = The group that has the highest frequency.

c) Identify the group in which the median would lie.

$$= \frac{56}{2} = 28^{\text{th}} \text{ Value.}$$

$$\text{Median Value} = \frac{\text{Total frequency} + 1}{2}$$

Add the frequency column until you reach the 28th value.

Median is in the group $20 < L \leq 30$

NOTE:

For grouped data, we can only calculate an estimate for each average as we do not know the exact values in each group.

Online clip

M287

Stem and leaf Diagrams



Component Knowledge

- Put data into a stem and leaf diagram
- Create a key to explain the diagram
- Find averages using a stem and leaf diagram

Key Vocabulary

Stem and leaf diagram	A diagram where each data value is split into a leaf and a stem
Ascending	From smallest to largest
Mean	A calculated central value of a set of numbers
Mode	The number which appears most often in a set of numbers
Median	The middle of a sorted list of numbers
Range	The difference between the lowest and highest values

Key Concepts

A **stem and leaf diagram** is a method of organising numerical data based on the place value of the numbers.

Each number is split into two parts:

- The first digit(s) form the stem
- The last digit forms the leaf

The leaf should only ever contain a single digit

How to set up a stem and leaf diagram

- Organise the data into ascending order, smallest to largest
- Determine how the numbers are split into 2 parts by writing a key for the stem and leaf diagram
- Write the values for the “stem” into the diagram
- Write the values for the “leaf” into the diagram

The key

A stem and leaf diagram must have a **key**. This explains how to convert the digits in the stem and leaf diagram into a single data point. Remember to include any units in the key if appropriate.

Key : 1 | 4 means 1.4kg

Key : 3 | 5 represents 35 years

Key : 1 | 9 represents 1.9kg

Boys Key : 1 | 4 represents 41 marks
Girls Key : 4 | 0 represents 40 marks

Example

A group of students are making models out of clay. The weight of each model is shown below. Draw a stem and leaf diagram.

1.5kg, 2.3kg, 1.6kg, 3.1kg, 3.1kg, 1.4kg, 2.5kg, 1.7kg, 1.8kg, 2.4kg

1) Order the numbers

1.4kg, 1.5kg, 1.6kg, 1.7kg, 1.8kg, 2.3kg, 2.4kg, 2.5kg, 3.1kg, 3.1kg

2) Split the numbers into two parts.

1.4kg splits into units (1) and tenths (4)

3) Put the values into the diagram and create a key

Key : 1	4 means 1.4kg
1	4 5 6 7 8
2	3 4 5
3	1 1

Dual Stem and Leaf diagrams

Comparing data sets is simplified by using a dual stem and leaf diagram which have two sets of data represented back to back.

For example, the two sets of data shown below could be combined together to form one dual stem and leaf diagram instead of having two separate diagrams

	Female		Male
0	6 7		0 1 4 5
1	2 4 8 8 9		1 0 2 3 4 4 8
2	4 5 5 5 6 6		2 2 7 7 9
3	0 1 2 3 3 3 6		3 0 0 0 3 6 8
4			4 0

Key : 1 | 2 means 12 Key : 1 | 0 means 10

Note the digits in the leaf for females is still in ascending order but from right to left, rather than left to right.

Female		Male
7 6	0	1 4 5
9 8 8 4 2	1	0 2 3 4 4 8
6 6 5 5 5 4	2	2 7 7 9
5 3 3 3 2 1 0	3	0 0 0 3 6 8
	4	0

Key : 3 | 1 | 4 represents
13 Female
14 Male

The data for the two classes is now much easier to compare and draw conclusions from

Averages from a stem and leaf diagram

The mode, median, mean and average can all be found from the data in a stem and leaf diagram

Key : 1 | 9 represents 1.9kg

1	9
2	2 8
3	1 4 4 0
4	5 8
5	1

From the stem and leaf we can see that 3.4kg is the mode and it appears the most in the diagram

The range is 3.2kg and the mean is 3.51kg

For the median we need to find the middle value. There are 10 values so to find the location of the median we do $(10 + 1) / 2 = 5.5$

We count 5 and a half places to find the median is 3.4kg

Or we can cross off from either side to find the middle number.

Online clips

M648, M210

Scatter Graphs



Component Knowledge

- Plot points on a scatter graph
- Describe the relationship between variables using a scatter graph
- Identify outliers on a scatter graph
- Draw and interpret a line of best fit

Key Vocabulary

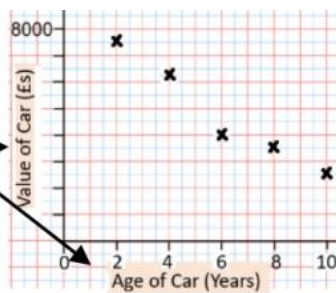
Origin	Where two axes meet on a graph
Outlier	A point that lies outside the trend of the graph
Relationship	The link between two variables
Correlation	The mathematical definition for the type of relationship between two variables
Line of best fit	A straight line on a graph that represents the data on a scatter graph
Interpret	Describe what the data is showing

Plotting a scatter graph

Age of car (years)	2	4	6	8	10
Value of car (£)	7500	6250	4000	3500	2500

The data forms information pairs for the scatter graph that you plot as coordinates e.g. (2, 7500).

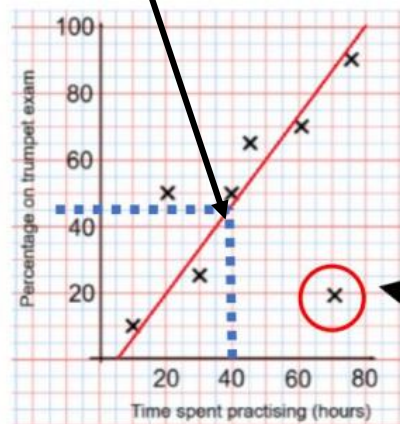
Make sure axes are clearly labeled and values are equally spaced



The line of best fit

We use the line of best fit to estimate other values.

E.g. 40 hours revising predicts 45% score on exam



We cannot use our line of best fit to predict information outside of our data range.

This point is an 'outlier' It doesn't fit the model and stands apart from the rest of the data

Types of Correlation- describes the relationship only.



Positive correlation
As one variable increases so does the other variable.



Negative correlation
As one variable increases the other variable decreases.



No correlation
There is no relationship between the two variables.

Online clips

M769, M596

Pie charts



Component Knowledge

- Calculate angles in a pie chart
- Draw a pie chart from a table
- Interpret pie charts using fractions
- Interpret pie charts using angles

Key Vocabulary

Angle	The amount of turn between 2 lines.
Pie chart	A chart that displays data proportionally.
Protractor	Equipment used to measure and draw angles

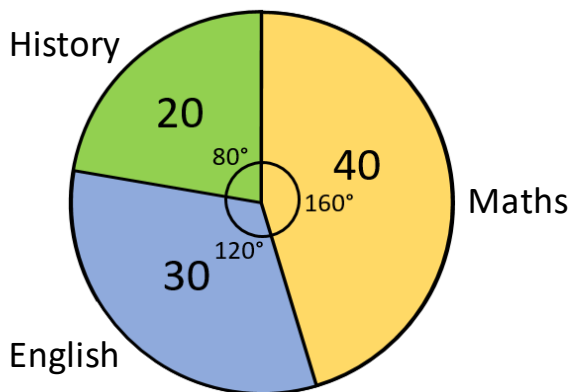
Drawing pie charts

How many degrees for one person? $\frac{360}{90} = 4^\circ$

$360 \div \text{total} = \text{degrees for one person}$. In this example one person is 4° .

Subject	Number of Students	Calculation	Angle
Maths	40	$40 \times 4^\circ$	160°
English	30	$30 \times 4^\circ$	120°
History	20	$20 \times 4^\circ$	80°
Total	90		360°

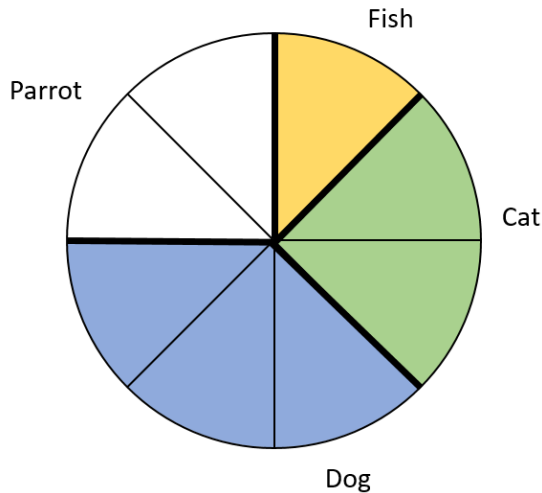
Multiply number of students by 4° to get the angle.



Draw the angles onto the pie chart. Label each part with what it is (subject in this example) and how many it represents (40 for Maths in this example).

Interpret pie charts (fractions)

A class of **32 students** were surveyed to find their **favourite pet**.
The **pie chart** shows the total answers. How popular was each animal?



The pie chart is split into 8 pieces,
so each sector is worth $\frac{1}{8}$ of $32 = 4$

$$\text{Fish: } \frac{1}{8} \text{ of } 32 = 4$$

$$\text{Cat: } \frac{2}{8} \text{ of } 32 = 8$$

$$\text{Dog: } \frac{3}{8} \text{ of } 32 = 12$$

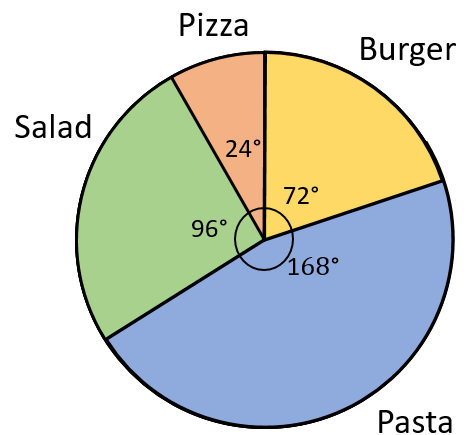
$$\text{Parrot: } \frac{2}{8} \text{ of } 32 = 8$$

Check that the totals add up to the original total in the question.
($4 + 8 + 12 + 8 = 32$)

Interpret pie charts (angles)

150 students were surveyed about their favourite food.

Favourite Food	Angle	Calculation	Frequency
Burger	72°	$\frac{72}{360} \times 150$	30
Pasta	168°	$\frac{168}{360} \times 150$	70
Salad	96°	$\frac{96}{360} \times 150$	40
Pizza	24°	$\frac{24}{360} \times 150$	10



To calculate the frequency from a pie chart when you are given the angle,
you do the opposite of what you do to calculate the angle.

$$\text{Angle} \div 360 \times \text{total frequency}$$

Online clips

M574, M165



Sampling

Component Knowledge

- Know the difference between random sampling and stratified sampling
- To know how to take a random sample
- To know how to calculate sample sizes for stratified sampling

Key Vocabulary

Qualitative data	Data collected that is described in words not numbers. e.g. race, hair colour, ethnicity.
Quantitative data	This is the collection of numerical data that is either discrete or continuous.
Population	This is the whole group you are collecting data from.
Sample	A sample is part of the whole population.

Simple Random Sampling

A simple random sample is when each member of the population under study has the same chance or probability of being selected for the sample.

An example of a simple random sample would be:

1. Assign a number to every member of the population
2. Randomly generate numbers using numbers from a hat or a computer calculator
3. Use the data from the corresponding members of the population

The following options are not random as not everyone has the same chance of being chosen:

- Choose the first 50 people who arrive at the office.
- Choose 50 people whose surname begins with J or T.
- List all the office workers in alphabetical order and choose every 5th name on the list.

Systematic sampling

- This is a very similar method to random sampling, but **the population would first be ordered** according to specific criteria such as listing names of people in the population in alphabetical order.
- The sample would be drawn by selecting every nth person. For example, **every 10th person in the list.**

Online clip

U162

A sample should be:

- fair and unbiased
- large enough in size to be representative of the whole population under study.

Stratified Sampling

A stratified sample involves grouping members of the population into classes before taking a proportionate sample from each class (e.g. grouped by age, language etc.)

To find the amount of people in each class we must do the following calculation $\frac{\text{Class size}}{\text{total population}} \times \text{sample size}$

Example

The table below shows the age group of the members of a tennis club.

Age Group	Junior	Adult	Senior
Number	320	500	130

Total population=

$$320 + 500 + 130 = 950$$

A stratified sample of 40 is to be taken. Calculate the number for each age group in the sample.

Junior

$$\frac{320}{950} \times 40 = 13.5 \approx 14 \text{ people}$$

Adult

$$\frac{500}{950} \times 40 = 21.1 \approx 21 \text{ people}$$

Senior

$$\frac{130}{950} \times 40 = 5.4 \approx 5 \text{ people}$$