Averages

Component Knowledge

- To understand and calculate the mode from a list.
- To understand and calculate the median from a list.
- To understand and calculate the mean from a list
- To calculate the upper and lower quartiles and understand that each is worth 25% of the data.
- To calculate the range and understand it is **not** an average.
- To calculate the interquartile range of a set of data.

Key Vocabulary

Data set	Collection of values that share a common relationship. This could be answers to a set question or information for a set objective.
Average	Is a value (or values) that is used to represent a whole data set
Mode	The most frequent value in a data set. It is a type of average. Modal is another word used more mode.
Median	The middle value of a data set, when ordered. It is a type of average.
Upper Quartile -UQ	Three-quarters of the way through the data set.
Lower Quartile- LQ	A quarter of the way through the data set.
Mean	A measure of the size of the data when shared out equally. It is a type of average.
Range	A value to show spread out a data set is. It can be used to describe how representative of the whole data set the average used is. IT IS NOT AN AVERAGE.
Interquartile range	The difference between the upper quartile (UQ) and the lower quartile (LQ). Calculated by UQ – LQ. Used to measure spread of data.

Averages

We use averages to summarise a whole data set in a single value/few values. We do this so we can interpret large data sets and also compare data sets more easily.

Mode- the most frequent value/ few values in a data set. There can also be no mode in a set of data.

Ex 1, find the mode:

blue red blue green blue blue
pink green blue red blue yellow

Blue is the mode.

Ex 2, find the mode:

9, 4, 3, 6, 9, 5, 2, 1, 8, 7. To make it easier, we can re-write these values in ascending(increasing) order. 1, 2, 3, 4, 5, 6, 7, 8, 9, 9. We can now see clearly **9 is the mode**.

Ex 3, find the mode: 9, 4, 3, 6, 9, 5, 2, 1, 8, 7, 3

Re-written 1, 2, 3, 3, 4, 5, 6, 7, 8, 9, 9 We can see **3 and 9 are the modal values**.

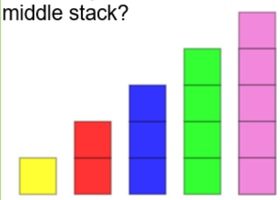
** We usually only have 1, 2 or 3 modal values**

Ex 4, find the mode: 4, 3, 6, 9, 5, 2, 1, 8, 7

Re-written 1, 2, 3, 4, 5, 6, 7, 8, 9 We can see there **are NO modal values**.

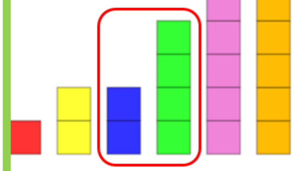
<u>Median-</u> the middle value in a data set, when in order. If there are 2 middle values, we find the midpoint between them.

How many blocks are in the middle stack?

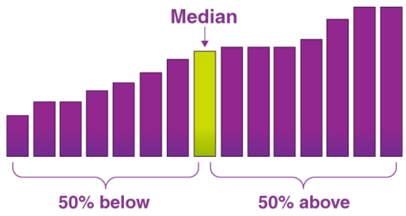


The middle stack has 3 blocks in

How many blocks are in the middle stack?



There is no "middle stack". We have to calculate the middle of 2 and 4. The middle would be 3.



Find the median of: ~~1~~, ~~3~~, ~~3~~, **6**, ~~7~~, ~~8~~, ~~9~~

Median = <u>6</u>

Find the median of: ~~1~~, ~~2~~, ~~3~~, **4**, **5**, ~~6~~, ~~8~~, ~~9~~

Median is the midpoint of 4 and 5 = <u>4.5</u>

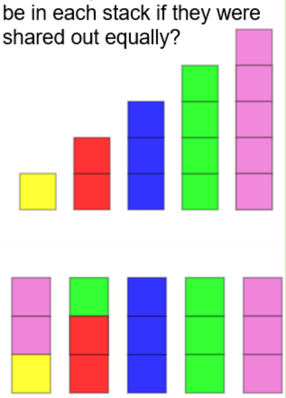
Find the median of the following set of numbers.

40 –2 10 40 –31 3 –34 –13 –10 1 30 16 –16

–34 –31 –16 –13 –10 –2 (1) 3 10 16 30 40 40

<u>Mean-</u> The mean is the size of each part when a quantity is shared equally. We can do this by adding all the values in the data set together and then dividing it equally between the number of values.

How many blocks would there be in each stack if they were shared out equally?



There would be three in each pile so the mean = 3

Example 1.
Find the mean of the following set of numbers.

19, 6, 17, 6

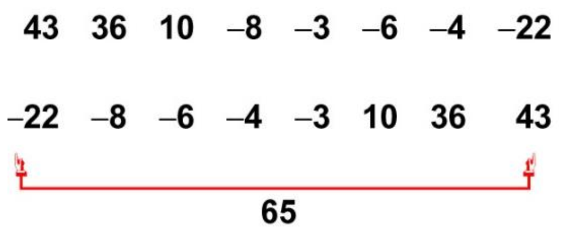
Solution.
To find the mean divide the sum of the numbers by the number of numbers.

$$\frac{\text{Sum of numbers}}{\text{Number of numbers}} = \frac{19+6+17+6}{4} = \frac{48}{4} = 12$$

There are 4 values in the data set so we are dividing by 4.

Range- the range shows how spread out the data is. It is useful to order the data when finding the range. The smaller the range, the more consistent the data.

E.g. Find the range of the following numbers



Range =43- -22 = 65

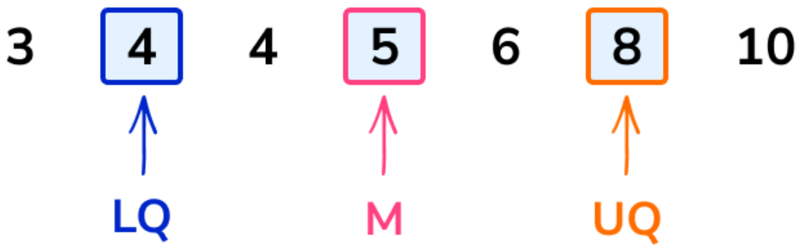
Online Clips

M841, M934, M940, M328

Quartiles – As well as the median value and range, it is sometimes useful to know the upper and lower quartiles when dealing with extreme values. Each quartile is worth 25% of the data.

Example

Find the interquartile range of the following data.



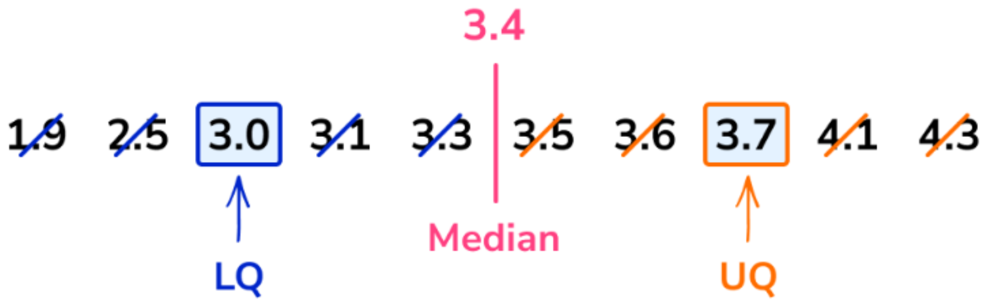
Here, $IQR = UQ - LQ = 8 - 4 = 4$

The interquartile range (IQR) measures the spread of the central 50% thereby avoiding any extreme values. It is therefore more representative of the spread of the data. It can show how consistent a set of data is.

Example 2

The data below shows the birth weights of 10 babies

3.3, 3.7, 2.5, 3.5, 3.0, 4.3, 3.1, 4.1, 1.9, 3.6



$$IQR = UQ - LQ = 3.7 - 3.0 = 0.7kg$$

Cumulative Frequency

Component Knowledge

- To be able to complete a cumulative frequency table.
- To be able to plot a cumulative frequency curve.
- To be able to calculate the median from the curve.
- To be able to calculate the inter-quartile range from the curve.

Key Vocabulary

Frequency	The number of times a data value occurs.
Cumulative frequency	The sum of frequencies to a certain point.
Ogive	A curved graph.
Median	The middle value when in order when in ascending order.
Quartile	The set of values which has three points dividing the data set into four identical parts.
Upper quartile	The value under which 75% of data points are found when arranged in increasing order.
Lower quartile	The value under which 25% of data points are found when arranged in increasing order.
Inter quartile range	The difference between the upper quartile and the lower quartile.

Completing a cumulative frequency table

The table shows information about the time taken to complete a puzzle.

Time	Frequency	Cumulative frequency
$0 < t \leq 10$	3	3
$10 < t \leq 20$	11	3 + 11 = 14
$20 < t \leq 30$	15	3 + 11 + 15 = 29
$30 < t \leq 40$	27	3 + 11 +15 +27 = 56
$40 < t \leq 50$	16	3 + 11 + 15 + 27 + 16 = 72
$50 < t \leq 60$	8	3 + 11 + 15 + 27 + 16 + 8 =80

To calculate the cumulative frequency, add the previous frequencies together and then add the current frequency.

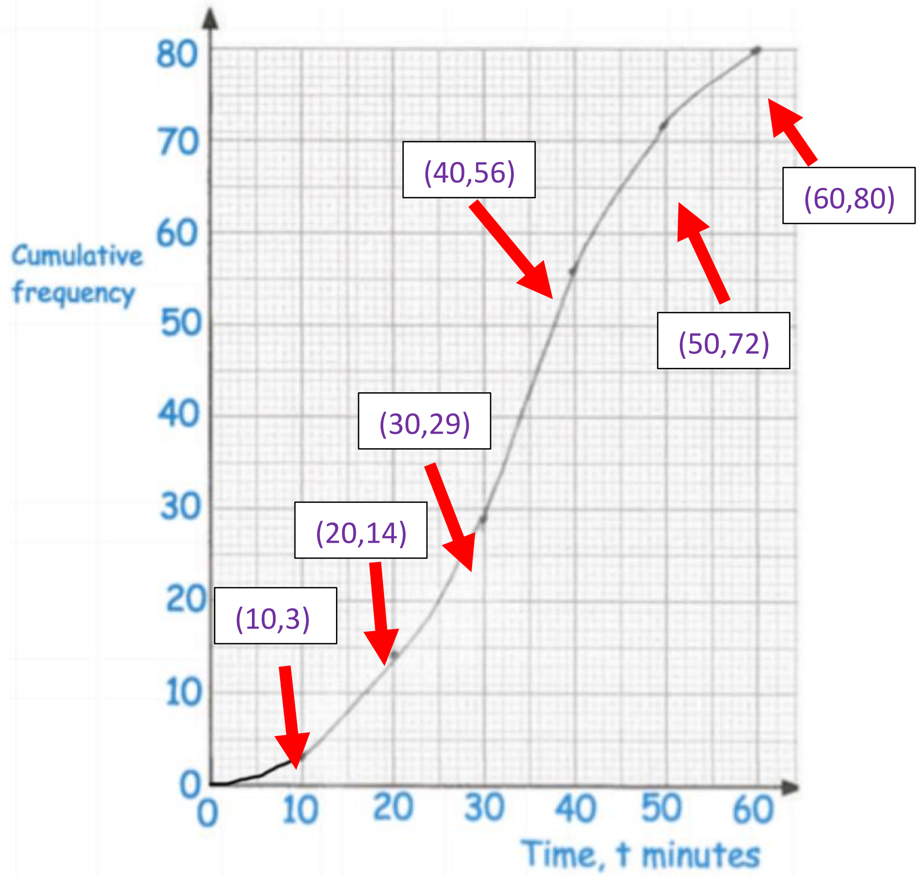
TOP TIPS FOR PLOTTING A CUMULATIVE FREQUENCY CURVE

1. Plot the cumulative frequency on the y-axis
2. Plot the time on the x-axis
3. Plot each point at the upper class boundary

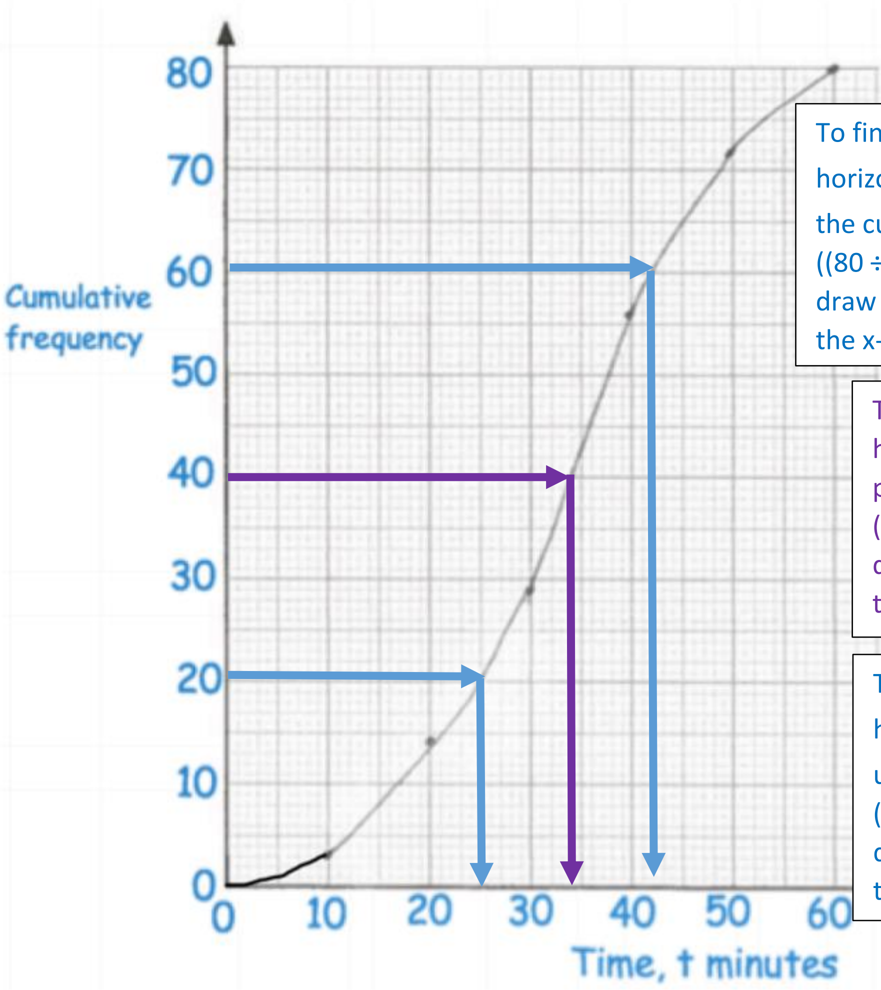
Plotting and drawing a cumulative frequency curve

Time	Frequency	Cumulative frequency
$0 < t \leq 10$	3	3
$10 < t \leq 20$	11	3 + 11 = 14
$20 < t \leq 30$	15	3 + 11 + 15 = 29
$30 < t \leq 40$	27	3 + 11 +15 +27 = 56
$40 < t \leq 50$	16	3 + 11 + 15 + 27 + 16 = 72
$50 < t \leq 60$	8	3 + 11 + 15 + 27 + 16 + 8 =80

Plot this point at the coordinate (10,3) as 10 is ***upper class boundary*** and 3 is the ***cumulative frequency.***



Calculating the median and inter-quartile range from a cumulative frequency curve



To find the upper quartile draw a horizontal line from $\frac{3}{4}$ of the way up the cumulative frequency axis ((80 ÷ 4) x 3 = 60) to the curve and then draw a vertical line from the curve to the x- axis. Upper quartile = 42

To find the median draw a horizontal line from the half way point of cumulative frequency axis (80÷2 = 40) to the curve and then draw a vertical line from the curve to the x- axis. Median = 34

To find the lower quartile draw a horizontal line from $\frac{1}{4}$ of the way up the cumulative frequency axis (80 ÷ 4 = 20) to the curve and then draw a vertical line from the curve to the x- axis. Lower quartile = 25

Inter quartile range = Upper quartile – Lower quartile

= 42 – 25

= 17

Online clips

U182, U642

Box Plots

Component Knowledge

- Plot Box Plots from lists of data
- Interpret key information from box plots
- Compare data using box plots

Key Vocabulary

Box Plot	A chart that displays the minimum, maximum, lower quartile and upper quartile for a set of data.
Upper Quartile (UQ)	This number that is in the middle of the upper half of the data at $\frac{3}{4}$.
Lower Quartile (LQ)	This number that is in the middle of the lower half of the data at $\frac{1}{4}$.
Inter-Quartile Range (IQR)	The difference between the upper and lower quartile containing the middle 50% of the data.
Median	The middle value for a set of data after the values have been put in order.
Range	The difference between the maximum and minimum value.
Compare	Analyse the differences and similarities for two or more things.

A box plot is a way of illustrating key information about a set of data. They are also very useful for comparing the distributions of multiple sets of data (e.g. boy vs girls). To construct a box plot you need five key pieces of information:

- The minimum value
- The lower quartile
- The median
- The upper quartile
- The maximum value

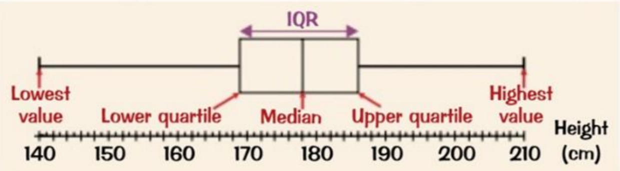
Box Plots – Key Information

1) The lower quartile Q_1, the median Q_2 and the upper quartile Q_3 are the values 25% (¼), 50% (½) and 75% (¾) of the way through an ordered set of data.
So if a set of data has n values, you can work out the positions of the quartiles using these formulas:

Q_1: (n + 1)/4 Q_2: (n + 1)/2 Q_3: 3(n + 1)/4

2) The INTERQUARTILE RANGE (IQR) is the difference between the upper quartile and the lower quartile and contains the middle 50% of values.

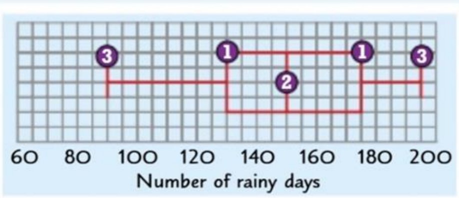
3) A box plot shows the minimum and maximum values in a data set and the values of the quartiles. But it doesn't tell you the individual data values.



EXAMPLE:

This table gives information about the numbers of rainy days last year in some cities.
On the grid below, draw a box plot to show the information.

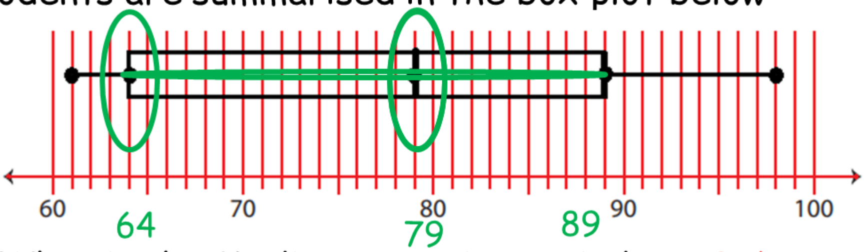
Minimum number	90
Maximum number	195
Lower quartile	130
Median	150
Upper quartile	175



1. Mark on the quartiles and draw the box.
2. Draw a line at the median.
3. Mark on the minimum and maximum points and join them to the box with horizontal lines.

Interpreting Box Plots

The gestation period for 28 different species of rodents are summarised in the box plot below



What is the Median gestation period? 79 days

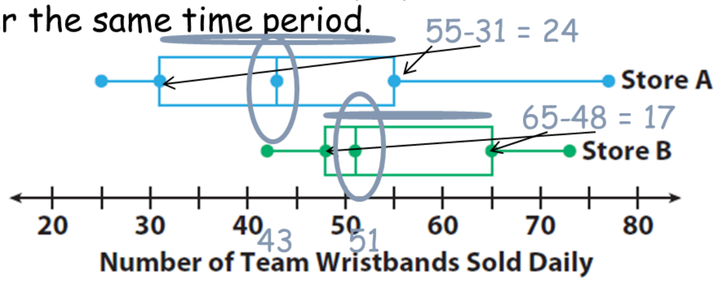
How many rodents were pregnant for less than 79 days? Median shows $\frac{1}{2}$ of them, $28 \div 2 = 14$ species

What is the IQR of gestation period for the sample of rodents? $IQR = Q_3 - Q_1 = 89 - 64 = 25$ days

How many rodents were pregnant for less than 64 days? Q_1 shows $\frac{1}{4}$ of them, $28 \div 4 = 7$ species

Using Box Plots to Compare Information

The box plots show the distribution of the number of team wristbands sold daily by two different stores over the same time period.



Compare the sales in store A with those in store B.

Store B has a larger median.

The median number of wristbands sold in Store A is 43, the median number of wristbands sold in Store B is 51.

On average Store B sells more wristbands per day.

Store A has a larger Inter Quartile Range.

The IQR of wristbands sold in Store A is 24, the IQR of wristbands sold in Store B is 17.

Generally the number of wristbands sold in Store A is more varied.

When comparing data sets using box plots the main things we need to discuss are the median and the inter-quartile range

Online clips

U879, U837, U507

Box plots and cumulative frequency

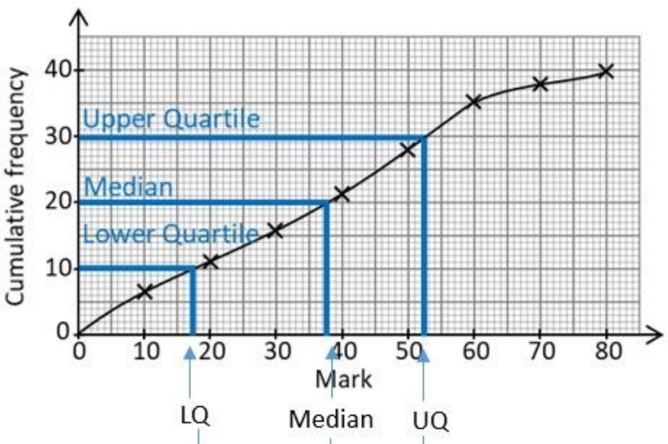
Component Knowledge

- Identify LQ, Median and UQ from a cumulative frequency graph
- Draw a box plot from a cumulative frequency graph

Key Vocabulary

Cumulative frequency	A graph that represents the running total of frequencies for each value in a data set. The graph is always a curve.
Box plot	A graph summarising a set of data. The shape of the boxplot shows how the data is distributed and it also shows any outliers.
Median	The value of the middle item of data when all the data is arranged in order
Lower Quartile (LQ)	The value under which 25% of data points are found when they are arranged in increasing order
Upper Quartile (UQ)	The value under which 75% of data points are found when arranged in increasing order

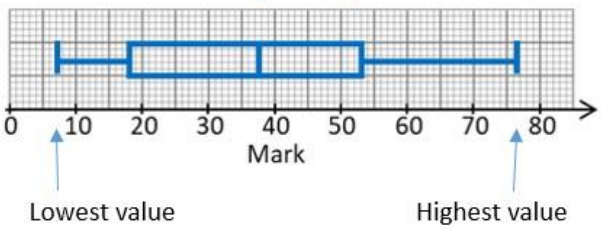
The cumulative frequency graph below shows the results of a Year 8 Maths test.
40 students sat the test.
The highest mark in the class with 76.
The lowest mark was 7.
A) Find the Median, Upper Quartile and Lower Quartile from the graph.



The Upper Quartile (UQ) is the 30th value.
The Median is the 20th value.
The Lower Quartile (LQ) is the 10th value.

Draw a line from the y axis until it meets the cumulative frequency curve. Then draw a line down until it meets the x axis. Read off the values from the x axis.
The UQ is 52
The Median is 37
The LQ is 17

B) Combine all this data to create a box plot.



The LQ, Median and UQ are the same as they are from the cumulative frequency above the box plot. The lowest and highest values are in the question.

Online clips

U642, U879, U507

Probability

Component Knowledge

- Understand what probability shows
- Understand probability notation
- Write a probability of a single event

Key Vocabulary

Probability	The mathematical chance, likelihood, of an outcome happening
Event	The “thing” that is being completed/done/observed/counted
(Event) Outcome	What happens when the event is performed
Probability scale	A numerical scale from 0 to 1, with 0 being an impossible outcome and 1 being an outcome certain to happen
Mutually exclusive (event) outcomes	When outcomes cannot happen at the same time eg being an adult and being a child, you **cannot be both**
Exhaustive (event) outcomes	When a set of outcome cover all possibility with no gaps eg it snowing and it not raining

Probability:

The probability of an (event) outcome A, happening is

$$P(outcome\ A) = \frac{number\ of\ ways\ outcome\ A\ can\ happen}{number\ of\ ways\ any\ outcome\ can\ happen}$$

e.g. the probability of rolling a number 4 on a regular 6 sided dice

$Outcome\ "4": \mathbf{4},\ so\ \ \mathbf{1\ option}$

$$P(roll\ a\ 4) = \frac{1}{6}$$

$All\ possible\ outcomes: \mathbf{1, 2, 3, 4, 5}\ or\ \mathbf{6},\ so\ \ \mathbf{6\ possibilities\ altogther}$

e.g. the probability of rolling a number greater than 4 on a regular 6 sided dice

$Outcomes\ "greater\ than\ 4": \mathbf{5}\ or\ \mathbf{6},\ so\ \ \mathbf{2\ options}$

$$P(roll\ a\ number\ greater\ than\ 4) = \frac{2}{6}$$

$All\ possible\ outcomes: \mathbf{1, 2, 3, 4, 5}\ or\ \mathbf{6},\ so\ \ \mathbf{6\ possibilities\ altogther}$

Online clips

M655, M941, M938, M755

Tree diagrams – independent

Component Knowledge

- Fill in missing values on a tree diagram
- Complete a tree diagram
- Find probabilities from a tree diagram

Key Vocabulary

Independent	An event that is not affected by other events
Probability	The chance that something happens
Event	One (or more) outcomes of an experiment
Outcome	A possible result of an experiment
Tree diagram	A diagram of lines connecting nodes, with paths that go outwards and do not loop back

Key Concepts

Independent events are events which do not affect one another.

Eg – replacing a counter before taking another from a bag

Probabilities on each set on branches add up to 1.

Probabilities can be written as fractions or decimals.

Probability Rules

The AND rule for probability states that the probability of A and B is the probability of A x the probability of B

The OR rule for probability states that the probability of A or B is the probability of A + the probability of B

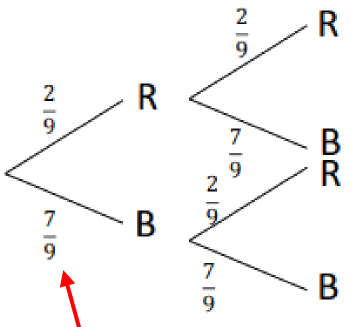
Example

There are red and blue counters in a bag.

The probability that a red counter is chosen is 2/9.

A counter is chosen and replaced, then a second counter is chosen.

Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:

$$\frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$$

Prob of two blues :

$$\frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$$

Prob of same colours:

$$\frac{4}{81} + \frac{49}{81} = \frac{53}{81}$$

Note – the probability of a blue counter is found by doing 1 – 2/9 to give 7/9

Online clips

U558

Tree diagrams - dependent

Component Knowledge

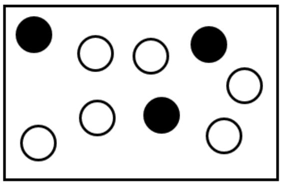
- Draw a probability tree for dependent events
- Calculate probabilities from a dependent event tree diagram

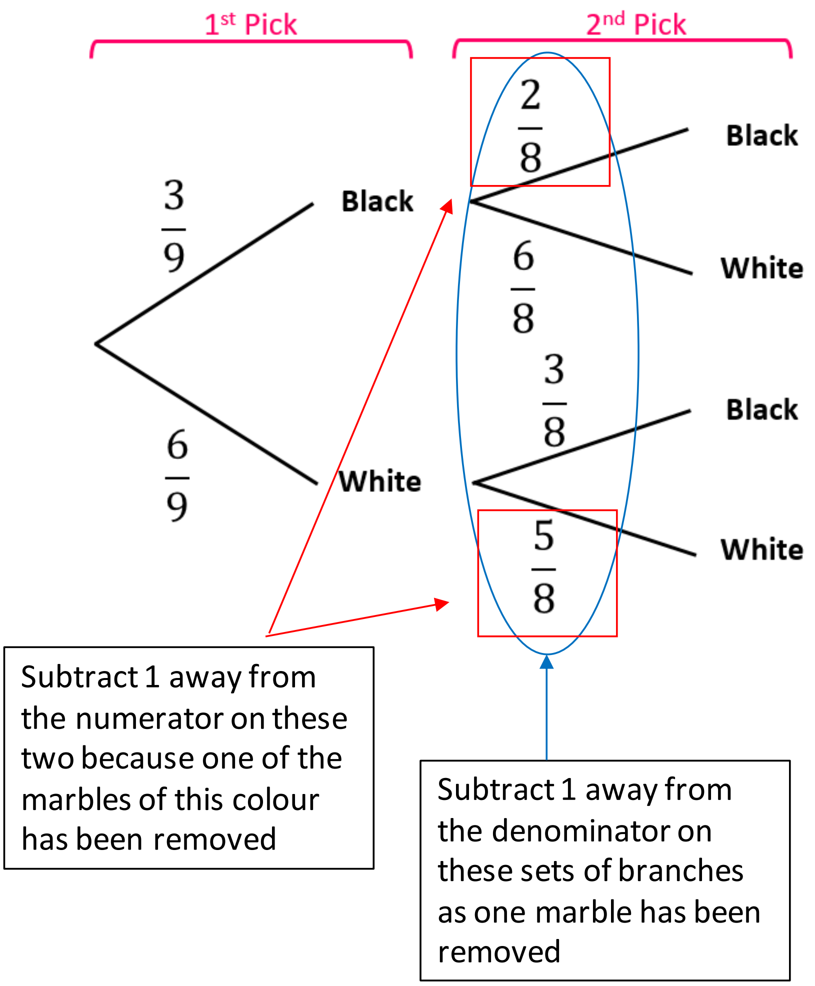
Key Vocabulary

Probability	The chance that something will happen
Event	The outcome of a probability
Tree diagram	Tree diagrams show all the possible outcomes of an event and helps to calculate their probabilities. Each set of branches must add up to 1.
Dependent	The outcome of a previous event does influence/affect the outcome of a second event.
Outcome	The result of a single performance of an experiment
AND rule	The outcome has to satisfy both conditions at the same time. Multiply the probabilities together.
OR rule	The outcome has to satisfy one condition, or the other, or both. Add the probabilities together.

Dependent tree diagrams

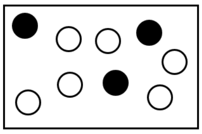
There are black and white marbles in the box. One is picked – **and not replaced** – then another is picked. Draw a probability tree to show this information.

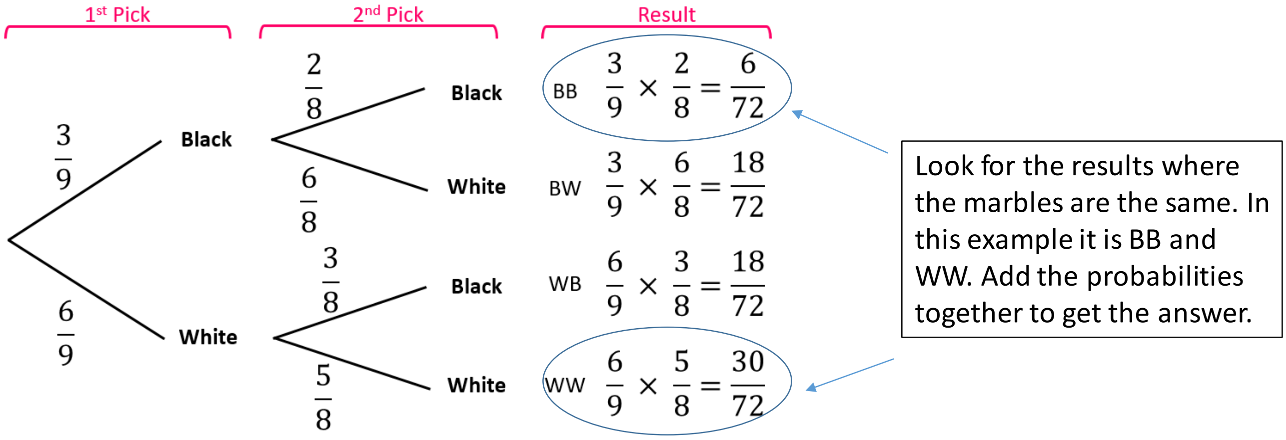




Dependent tree diagrams – calculating probabilities

There are black and white marbles in the box. One is picked – **and not replaced** – then another is picked. Draw a probability tree to show this information.

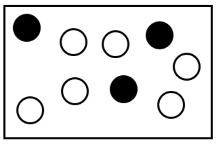


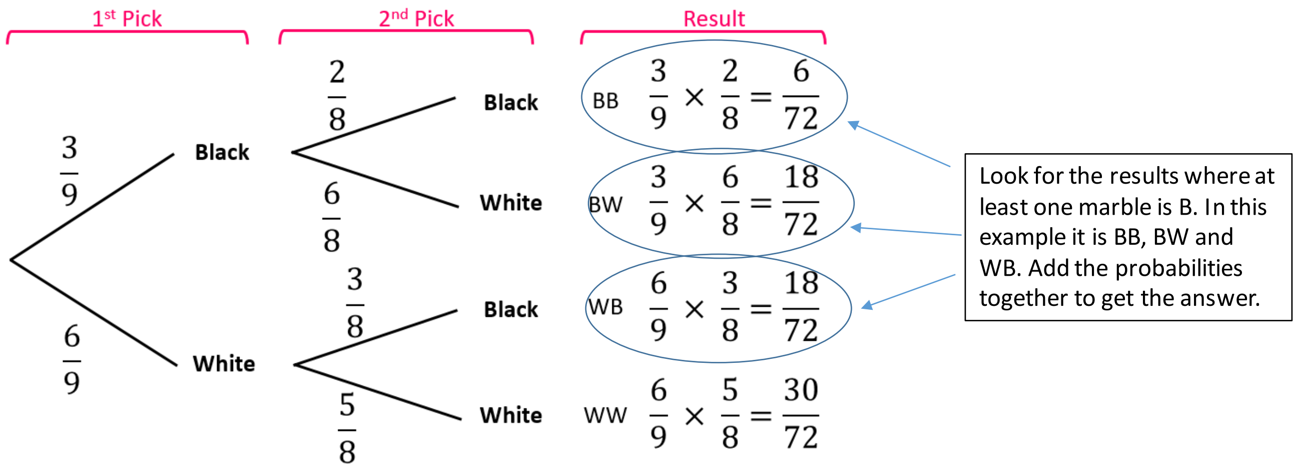


What is the probability two marbles of the **same colour** are picked?

P(Same colour) = $\frac{6}{72} + \frac{30}{72} = \frac{36}{72}$

There are black and white marbles in the box. One is picked – **and not replaced** – then another is picked. Draw a probability tree to show this information.





What is the probability **one or more** black marbles are picked?

P(1+ Black) = $\frac{6}{72} + \frac{18}{72} + \frac{18}{72} = \frac{42}{72}$

Online clips

U729

Inequalities

Component Knowledge

- Understand and use inequality notation
- Represent the solution set of an inequality on a number line
- Decide whether a number satisfies an inequality
- Form an inequality from a question and solve it

Key Vocabulary

Inequality	An inequality shows that two quantities are (may) not be equal
Less than	This is shown by the symbol $<$
Less than or equal to	This is shown by the symbol $\leq$
Greater than	This is shown by the symbol $>$
Greater than or equal to	This is shown by the symbol $\geq$
Integer	A whole number

Notation

$x > 2$ means x **is greater than 2**

$x < 3$ means x **is less than 3**

$x \geq 1$ means x **is greater than or equal to 1**

$x \leq 6$ means x **is less than or equal to 6**

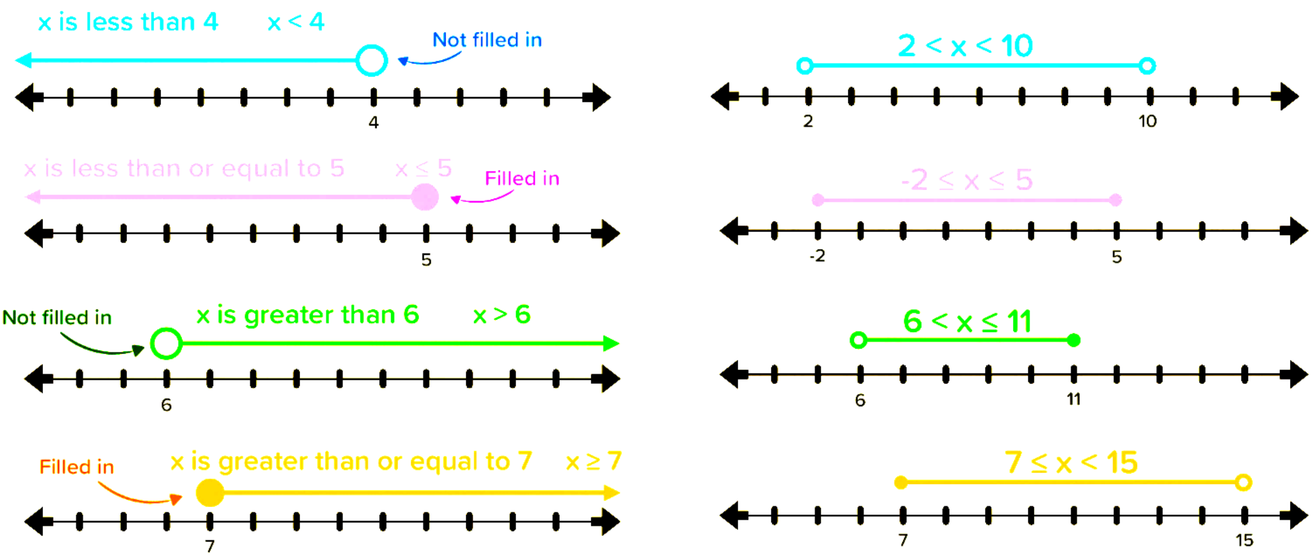
Examples:

$x \geq 1$ is **true** for $x = 6, 2.5$ and 1

$x < 5$ is **false** for $x = 10, 5.05$ and 5

The set of *integers* which **satisfy** $-2 \leq x < 3$ is $\{-2, -1, 0, 1, 2\}$

The set of numbers *satisfying* an inequality can be *represented* on a number line:



Inequalities can be **solved** by the same method as used for equations:

a) $x - 7 \leq 12$

b) $5y > 40$

c) $\frac{b}{3} \geq -2$

One-step solution

$$x - 7 \leq 12 \quad (+7)$$
$$x \leq 19$$

$$5y > 40 \quad (\div 5)$$
$$y > 8$$

$$\frac{b}{3} \geq -2 \quad (\times 3)$$
$$b \geq -6$$

Inverse operation

a) $5(x - 1) < 3.5$

$\frac{b}{6} + 2 \geq 1$

Two-step solution

$$5(x - 1) < 3.5 \quad (\div 5)$$
$$x - 1 < 0.7 \quad (+1)$$
$$x < 1.7$$

$$\frac{b}{6} + 2 \geq 1 \quad (-2)$$
$$\frac{b}{6} \geq -1$$
$$b \geq -6$$

Make sure you write an inequality symbol

Online clips

M384, M118

Plotting straight line graphs

Component Knowledge

- Use substitution to create a table of values
- Plot the coordinates from a table of values to draw a straight line graph

Key Vocabulary

Plot	To draw on a graph or map
Equation	An equation says that two things are equal
Coordinates	A set of values that show an exact position
Gradient	How steep a line is
Y intercept	Where a line crosses the y axis
Linear equation	An equation that makes a straight line when it is plotted

What is a straight line graph?

A straight line graph is a visual representation of a linear function. It has a general equation of: $y = mx + c$

Where m is the gradient of the line and c is the y intercept

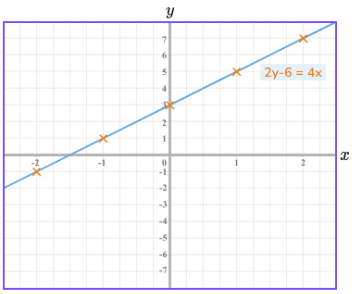
Plotting a line not in the form y = mx + c

Your equation will not always be in the correct form so may need rearranging before a table of values can be created.

For example, the line $2y - 6 = 4x$ would need to be rewritten to give $y = 2x + 3$.

This is an example of a table of values for this equation and the graph of the equation.

x	-2	-1	0	1	2
y	-1	1	3	5	7



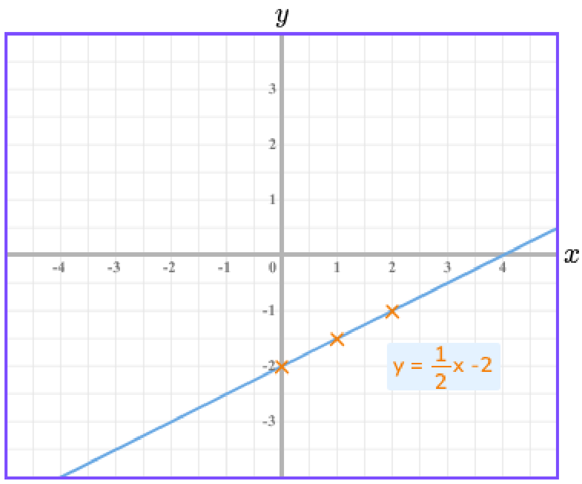
Plotting a straight line graph

In order to plot straight line graphs we need to substitute values for x into the equation for the graph and work out the corresponding values for y.

We use a table of values like the one below to do this.

x	-3	-2	-1	0	1	2	3
y							

Once you have your coordinates, plot them on the graph and use a ruler to draw a straight line through them. You can extend the line past the points you have plotted like in this example



Online clips

U741

Graphing Inequalities

Component Knowledge

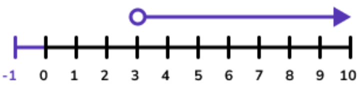
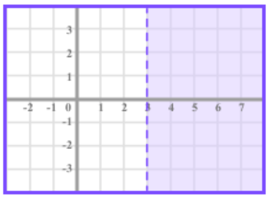
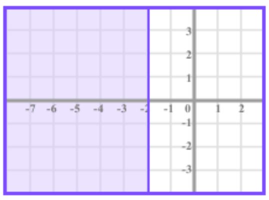
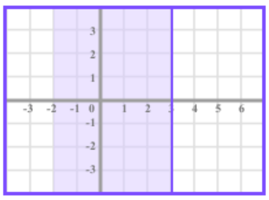
- Plot linear inequalities with dotted or solid lines.
- Shade graph regions.

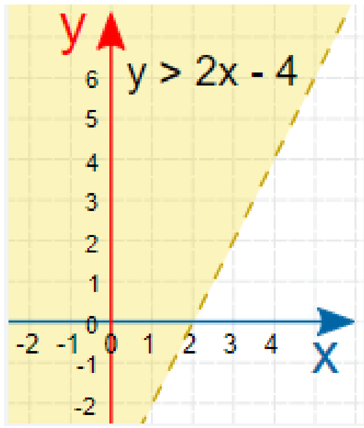
Key Vocabulary

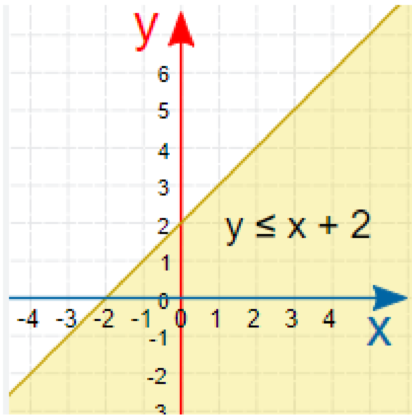
Linear	Relating to a graph that is a line.
Inequality	The relationship between two expressions that are not equal.

Representing an inequality on a graph is very similar to representing an equation on a graph. We plot a straight line but then we shade the region that satisfies the inequality.

- A dashed line is used for an inequality containing < and >
- A sold line is used for an inequality containing ≤ and ≥

On a number line	On a graph
$x > 3$ 	$x > 3$ We use a dashed line for $x = 3$ and can shade the region required to the right of the line. 
$x \leq -2$ 	$x \leq -2$ We use a solid line for $x = -2$ and can shade the region required to the left of the line. 
$-2 < x \leq 3$ 	$-2 < x \leq 3$ We can use a dashed line for $x = -2$ and a solid line for $x = 3$. We can shade the region required in between the lines. 

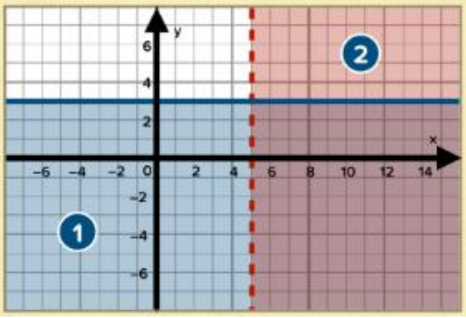


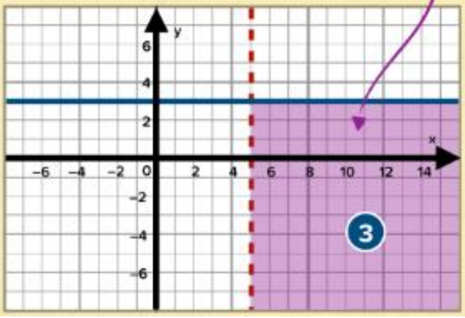


Example 1

Q Shade the region which satisfies the following inequalities: $y \le 3$ and $x > 5$

1. Indicate* the region which satisfies $y \le 3$
2. Indicate* the region which satisfies $x > 5$
3. The required region is where the two inequalities overlap

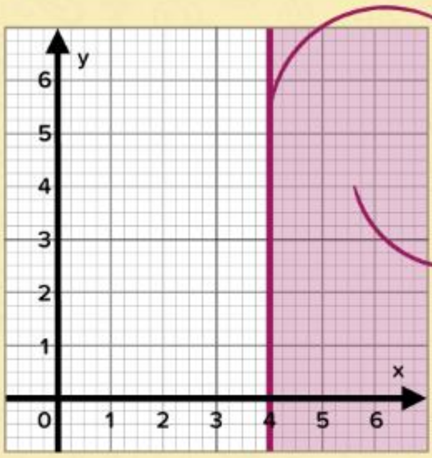




Example 2

! Remember

A linear inequality can be represented on a graph by a shaded or labelled region.



The equation of this line: $x = 4$

The inequality represented by the shaded region: $x \ge 4$ The values of the line **are** included.

! Remember

Strict inequality (< or >) → dashed line

Non-strict inequality (≤ or ≥) → solid line

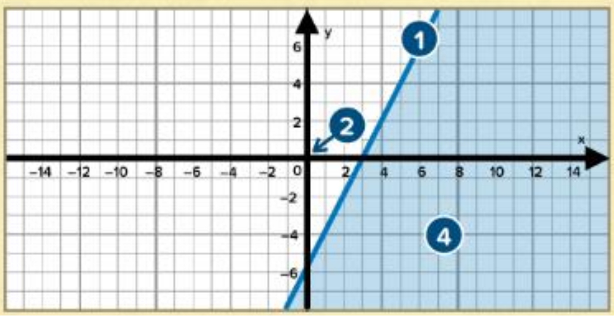
Example 3

Q Which region satisfies the inequality: $y \le 2x - 6$

1. Draw the line: $y = 2x - 6$ It is a solid line as the inequality is not strict (≤).
2. Choose a point to test. The point (0, 0) is the easiest option.
3. Substitute the point into the inequality

   $2x - 6 = 2 \times 0 - 6 = -6$

   $0 \le -6$ ✗ False
4. Shade the side which does not include the point



Online links

U747

Error Intervals

Component Knowledge

- To use understand how to round to different degrees of accuracy.
- To be able to write error intervals when rounding using correct inequality notation.
- To be able to write error intervals when rounding using correct inequality notation.

Key Vocabulary

Rounding	Rounding means making a number simpler but keeping its value close to what it was. The result is less accurate, but easier to use.
Accuracy	How close the rounded value is to the original value.
Place value	The value of the digit in a number
Lower bound	The smallest possible value that can be rounded to the number given.
Upper bound	The largest possible value the rounded value can take.
Truncation	Truncation comes from the word truncare, meaning "to shorten". The number is cut off at a certain point.
Inequality notation	Symbols used to describe the relationship between two expressions that are not equal to one another.

Inequality Notation

All error intervals look the same like this:

$$\_\_\_\_ \leq n < \_\_\_\_$$

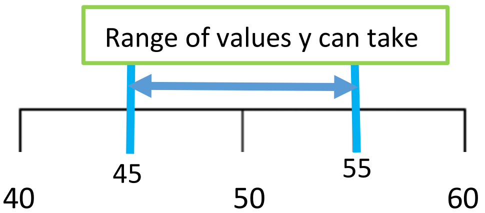
The value, n, can be greater or equal to this number.

The value, n, can only be less than this number but we use it to make any calculations easier to perform, should we need to.

Error intervals- rounding according to place value

Example 1- Frank rounds a number, y, to the nearest ten. His result is 50 Write down the error interval for y.

Begin by finding the ten, in this case, greater than and less than 50.



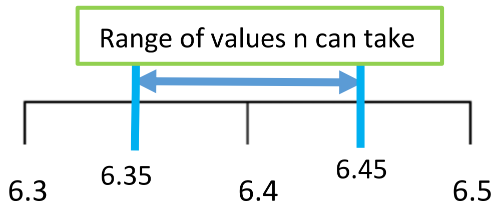
The midpoint between 40 and 50 is 45. This is the lower bound.

The midpoint between 50 and 60 is 55. This the upper bound (this can never = 55 but can be as large as 54.9999999….. 55 is easier to calculate with. Additionally, we use < as well.

The answer is $45 \leq y < 55.$

Example 2- Freya rounds a number, n, to one decimal place. Her result is 6.4 Write down the error interval for n.

Begin by finding the tenth, in this case, greater than and less than 6.4. (**Note: 1dp = tenths column.**)



The midpoint between 6.3 and 6.4 is 6.35. This is the lower bound.

The midpoint between 6.4 and 6.5 is 6.45. This the upper bound (this can never = 6.45 but can be as large as 6.49999999….. 6.45 is easier to calculate with. Additionally, we use < as well.

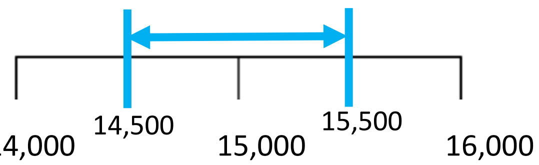
The answer is $6.35 \leq n < 6.45.$

Error intervals- rounding according to significant figures

Depending on the size of the number, the rounding will change when rounding to significant figures. Rounding like this keeps all numbers rounded to the same degree of accuracy relative to the size of the number.

Example 3- A number, g, is 15,000 when rounded to 2 significant figures. Write down the error interval.

Begin by finding the place value of the 2nd significant figure, in this case, this is 5000. This means we are rounding to 2 sig figs = rounding to nearest thousand.



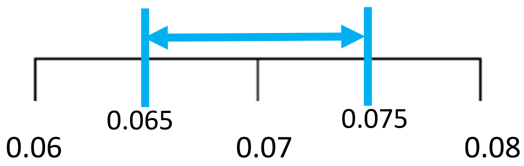
The midpoint between 14,000 and 15,000 is 14500. This is the lower bound.

The midpoint between 15,000 and 16,000 is 15,500. This the upper bound.

The answer is $14{,}500 \leq g < 15{,}500.$

Example 4- A number, x, is 0.07 when rounded to 1 significant figure. Write down the error interval.

Begin by finding the place value of the 1st significant figure, in this case, this is 0.07. This means we are rounding to 1 sig fig =rounding to nearest hundredth.



The midpoint between 0.06 and 0.07 is 0.065. This is the lower bound.

The midpoint between 0.07 and 0.08 is 0.075. This the upper bound.

The answer is $0.065 \leq x < 0.075.$

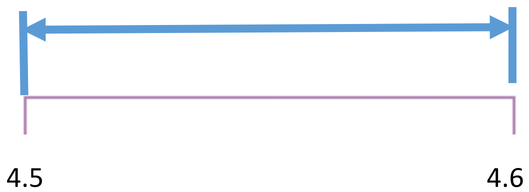
Error intervals- truncation

Be careful when reading error interval questions as truncating is not rounding like place value. The number has been "chopped", which means the value given **IS THE LOWER BOUND.** It commonly applies to decimals.

Example 5- State the error interval of 4.5 when it has been truncated to 1 decimal place.

Begin by finding the tenth, in this case, greater than 4.5. (**Note: 1dp = tenths column.**) This is the upper bound.

Remember: the value cannot equal 4.6!



The answer is $4.5 \leq n < 4.6.$

Online clip

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Bounds

Component Knowledge

- To be able to describe the range of values a rounded number make take.
- To use error intervals to calculate lower and upper bounds of calculations involving rounded numbers.

Key Vocabulary

Error interval	The range of values a rounded number can take.
Bound	The range of values a rounded answer to a calculation can take.
Lower bound	The lowest number a rounded value can take in a calculation.

Bounds – When using bounds you must work out the error intervals first before calculations.

Remember to use inequality signs in your answers- the lower bound inequality can be 'equal to', but the upper bound cannot be 'equal to'.

Example: Jack is 1.8 m tall (rounded to the nearest 10 cm). Ella is 1.63 m tall (rounded to the nearest cm). What is the smallest possible difference in their heights?

Step 1- find the error intervals for both values.
ALWAYS show this clearly.

Rounded value	Lower bound	Upper bound
1.8m = 180cm (to 10cm)	175cm	185cm
1.63m= 163cm (to 1cm)	162.5cm	163.5cm

Step 2- We are looking for the smallest possible difference, which means we use the numbers with the smallest difference from the table for Jack and Ella.

This is 175cm for Jack and 163.5cm for Ella.

The difference is 175cm – 163.5cm = 11.5cm

Example: V = IR

I = 5.92 correct to 2 decimal places R = 12.356 correct to 3 decimal places.

Work out the upper bound for V. Give your answer to 3 decimal places.

Step 1- find the error intervals for both values.

Rounded value	Lower bound	Upper bound
I = 5.92 (to 2dp)	5.915	5.925
R = 12.356 (to 3dp)	12.3555	12.3565

Step 2- We are looking for the largest possible value, which means we use the upper bound for I and the upper bound for R.

V = 5.925 x 12.3565 = 73.2122625

V = 73.212 (2dp)

Useful combinations

Operation	Minimum	Maximum
Addition (a + b)	$a_{min} + b_{min}$	$a_{max} + b_{max}$
Subtraction (a - b)	$a_{min} - b_{max}$	$a_{max} - b_{min}$
Multiplication (a x b)	$a_{min} \times b_{min}$	$a_{max} \times b_{max}$
Division (a ÷ b)	$a_{min} \div b_{max}$	$a_{max} \div b_{min}$

Online clips

U657, U587

Sampling

Component Knowledge

- Know the difference between random sampling and stratified sampling
- To know how to take a random sample
- To know how to calculate sample sizes for stratified sampling

Key Vocabulary

Qualitative data	Data collected that is described in words not numbers. e.g. race, hair colour, ethnicity.
Quantitative data	This is the collection of numerical data that is either discrete or continuous.
Population	This is the whole group you are collecting data from.
Sample	A sample is part of the whole population.

Simple Random Sampling

A simple random sample is when each member of the population under study has the same chance or probability of being selected for the sample.

An example of a simple random sample would be:

1. Assign a number to every member of the population

2. Randomly generate numbers using numbers from a hat or a computer calculator

3. Use the data from the corresponding members of the population

The following options are not random as not everyone has the same chance of being chosen:

- Choose the first 50 people who arrive at the office.
- Choose 50 people whose surname begins with J or T.
- List all the office workers in alphabetical order and choose every 5th name on the list.

Systematic sampling

- This is a very similar method to random sampling, but the population would first be ordered according to specific criteria such as listing names of people in the population in alphabetical order.
- The sample would be drawn by selecting every nth person. For example, every 10th person in the list.

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U162

A sample should be:

- fair and unbiased
- large enough in size to be representative of the whole population under study.

Stratified Sampling

A stratified sample involves grouping members of the population into classes before taking a proportionate sample from each class (e.g. grouped by age, language etc.)

To find the amount of people in each class we must do the following calculation $\frac{\text{Class size}}{\text{total population}} \times \text{sample size}$

Example

The table below shows the age group of the members of a tennis club.

Age Group	Junior	Adult	Senior
Number	320	500	130

Total population=

320 + 500+ 130 = 950

A stratified sample of 40 is to be taken. Calculate the number for each age group in the sample.

Junior

$\frac{320}{950} \times 40 = 13.5 \approx 14 \text{ people}$

Adult

$\frac{500}{950} \times 40 = 21.1 \approx 21 \text{ people}$

Senior

$\frac{130}{950} \times 40 = 5.4 \approx 5 \text{ people}$

Capture-Recapture

Component Knowledge

- Apply the capture recapture method to estimate the size of a population.

Key Vocabulary

Sample	A selection of data from a larger group of data, (called the population.) A sample should be representative of the population, this means the sample and the population should have similar properties.
Population	The whole group from where the sample is taken.
Proportion	The size, number or amount of one thing or group as compared to the size, number or amount of another.

What is it? Capture recapture is a method used to estimate populations where it can be difficult to record all members of the population exactly e.g. animal populations.

The method:

1) Take a sample of the population.
2) Mark each item.
3) Put the items back into the population and ensure they are thoroughly mixed.
4) Take a second sample and count how many of your sample are marked.
5) The proportion of marked items in your sample should be the same as the proportion of marked items from the population in your first sample.

The Formula:

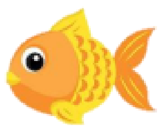
$$\frac{\textit{Size of first sample}}{\textit{Size of population}} = \frac{\textit{Number recaptured}}{\textit{Size of second sample}}$$

Example: **10** fish are caught in a lake, marked, and released back into the lake. A week later, **20** fish are caught and **4** are found to be marked. Estimated the number

X 2.5

$$\frac{10}{N} = \frac{4}{20}$$

X 2.5



N = 50

There are approximately 50 fish in the lake.

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U328