Bearings

Component Knowledge

- To be able to understand the 3 rules of bearings and use this to measure and draw bearings.
- To be able to use angle facts to find missing bearings.

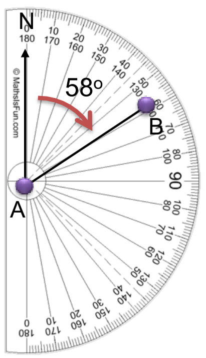
Key Vocabulary

Bearing	A measure of direction, it is used to represent the direction of one point relative to another. It is the angle in degrees measured clockwise from north. Always written in three-figures.
Protractor	The instrument used for measuring angles (measured in degrees).
Scale	Used to reduce real world dimensions to a useable size.

Measuring and drawing Bearings using a protractor:

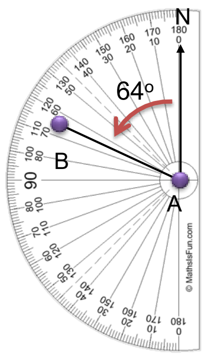
Bearings are measured and drawn

- From the **North** (N)
- **clockwise**
- are always written as **3 figures.**



Bearing = 058°

To measure a bearing greater than 180°, measure the angle anticlockwise and subtract from 360°.



Bearing = 360° – 64° = 296°

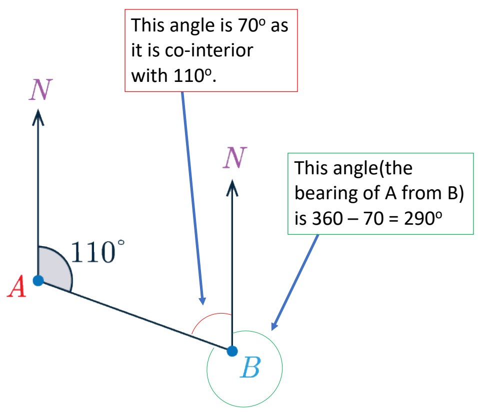
Be careful where a bearing is being measured from. If you were measuring the bearing of B from A your protractor would be on A.

Example: Bearings without a protractor

We are given the bearing of B from A. To calculate the area of A from B we can use angle facts.

Co-interior angles add up to 180°.

Angles at a point equal 360°.



Online clips

U525, U107

Bearings with Trigonometry

Component Knowledge

- To apply trigonometry to bearings problems using right-angled triangles.
- To apply trigonometry to bearings problems using any triangle.

Key Vocabulary

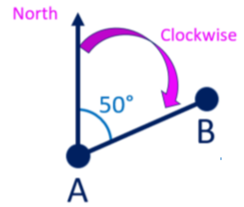
Bearing	Bearings are angles, measured clockwise from north.
Trigonometry	Branch of maths which calculates lengths and angles using specific ratios or formulae.

Important facts to remember:

Bearings are measured CLOCKWISE, from NORTH in 3 FIGURES.

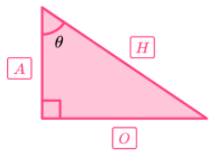
Write bearings with 3 figures

The bearing of B from A is 050°



Trigonometric ratios and formulae:

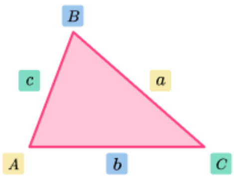
Right-Angled Triangles



$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A}$

Non-Right Angled Triangles



Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

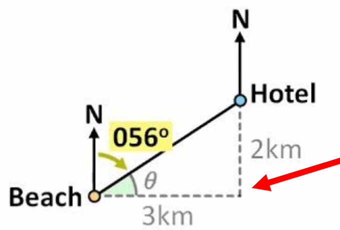
Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Bearings with Trigonometry- we will not be told to use trigonometry so we need to recognise this as part of the question. You should be thinking about triangles if you cannot directly solve the bearings problem.

Examples include:

A hotel is located 3km east and 2km north of the beach. Calculate the bearing of the hotel from the beach to the nearest degree.



This has created a right-angled triangle so we can use tan θ $= \frac{O}{A}$

$\tan(\theta) = \frac{2}{3}$

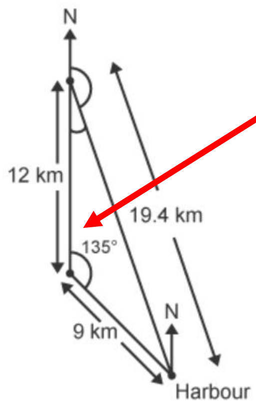
$\theta = \tan^{-1}\left(\frac{2}{3}\right) = 33.7°$

Bearing $= 90° - 33.7°$

$= 56.3°$

$= 056°$

A boat leaves the harbour and travels 9 km north west, then 12 km north. Calculate its distance from the harbour.



We need to sketch the problem first. We know this angle =135° (turn from NW to N).

The distance of the boat from the harbour is the missing side.

We know two sides and included angle, so we use the cosine rule.

$$a^2 = b^2 + c^2 - 2bcCosA$$

$$a^2 = 9^2 + 12^2 - 2 \times 9 \times 12 \times Cos(135)$$

$$a = \sqrt{377.74\ldots}$$

$$\underline{a = 19.4km(1dp)}$$

Online clip U164

Plotting quadratic graphs

Component Knowledge

- Create a table of values by substituting into a quadratic equation
- Plot coordinates and connect with a smooth curve
- Identify important parts of the graph

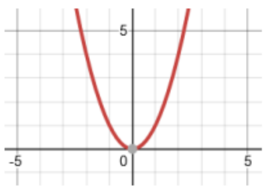
Key Vocabulary

Quadratic	Where the highest exponent of the variable (usually "x") is a square (^{2})
Roots	Where a function equals zero
Y Intercept	The point where a line or curve crosses the y axis of a graph
Turning Point	The point at which a graph changes direction
Line of Symmetry	A line that cuts something exactly in half

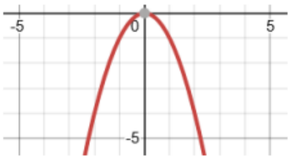
Key Concepts

A quadratic graph will always be in the shape of a parabola.

$y = x^2$



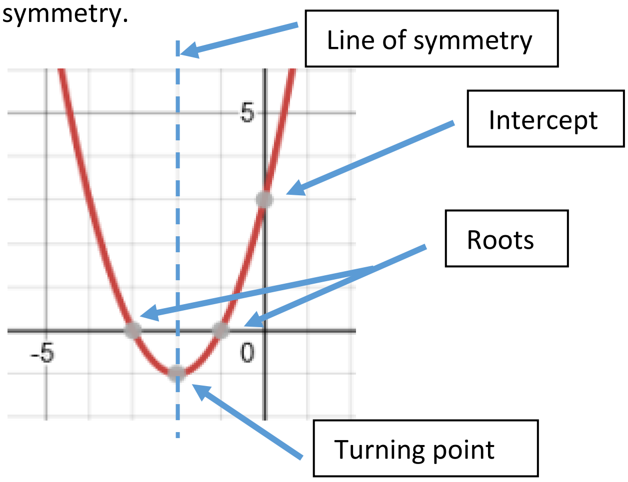
$y = -x^2$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.

Identifying important points

From the graph, you should be able to identify the roots, turning point, intercept and line of symmetry.



Plotting a quadratic graph

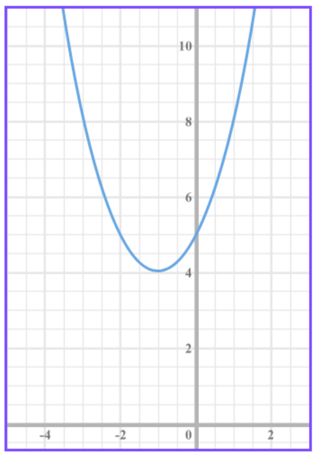
You can draw a table of values by substituting values for x into your equation to find the matching y value. These coordinates are then plotted and a smooth curve in drawn to connect the points.

For the equation $y = x^2 + 2x + 5$ the table of values is shown below and the graph opposite.

x	-3	-2	-1	0	1	2
y	8	5	4	5	8	13

$2^2 + (2 \times 2) + 5$
$= 13$

$(-3)^2 + (2 \times -3) + 5 = 8$ Remember $(-3)^2 = -3 \times -3 = 9$ not -9



Online clips U989, U667

Factorising Quadratics

Component Knowledge

- Be able to factorise a quadratic of the form $ax^2 + bx + c$ when $a = 1$
- Be able to factorise a quadratic of the form $ax^2 + bx + c$ when $a \neq 1$
- Factorise a quadratic using the difference of 2 squares
- To use factorising to solve a quadratic equation

Key Vocabulary

Quadratic expression	Equation of the form $ax^2 + bx + c$, where a, b and c are any form of number
Coefficient	Number of front of a letter, e.g. the coefficient of x^2 in the term $-5x^2$ is -5
Factor	A common number or letter that will divide into a term
Factorise	An expression written as a product of it's factors
Product	Multiplication of two or more values

Factorising when a = 1

Factorise

$$x^2 + 6x + 8$$

We need two numbers with a product of +8 and a sum of +6, so we list all of the products and check their sum

$1 \times 8 = 8$ $\quad 1 + 8 = 9$

$2 \times 4 = 8$ $\quad 2 + 4 = 6$

This is the correct product/sum pair

We then re-write the quadratic as

$$x^2 + 2x + 4x + 8$$

Factorising each half separately gives

$$x(x + 2) + 4(x + 2)$$

Taking out the common factor of the bracket then gives

$$(x + 2)(x + 4)$$

Difference of two squares

Look out for this specific case where

$$a^2 - b^2 = (a + b)(a - b)$$

Remember that this won't work if it contains + instead of −.

1) Factorise $x^2 - 25 = (x + 5)(x - 5)$ or $(x - 5)(x + 5)$

(Expanding gives $x^2 - 5x + 5x - 25 = x^2 - 25$.)

Sometimes there could be more than one variable (letter) in the expression.

2) $9x^2 - y^2 = (3x + y)(3x - y)$ or $(3x - y)(3x + y)$

3) $25c^2 - 16d^2 = (5c + 4d)(5c - 4d)$ or $(5c - 4d)(5c + 4d)$

Factorising when a ≠ 1

1) Factorise $2x^2 + 11x + 12$.

Here you need two numbers with a product of +24 (from +2 × +12) and a sum of +11.

The two numbers are +3 and +8.

Re-write the expression using your two numbers (in either order) to replace the middle term.

(Note that this time you don't put them straight into brackets!)

$2x^2 + 3x + 8x + 12$ or $2x^2 + 8x + 3x + 12$

Factorise a pair of terms at a time, by taking out common factors. (Make sure the 'introduced brackets' contain identical terms.)

$x(2x + 3) + 4(2x + 3)$ or $2x(x + 4) + 3(x + 4)$

Write down the 'repeated' bracket, then construct a second bracket using 'everything else'.

You then have:

$(2x + 3)(x + 4)$ or $(x + 4)(2x + 3)$

Factorising when a ≠ 1 (involving negatives)

Factorise

$2x^2 - 5x - 12$

We need two numbers with a product of +24 and a sum of +11,

so we list all of the products and check their sums

$2 \times 12 = 8 \quad 2 + 12 = 9,\ -2 + 12 = 10,\ 2 + (-12) = -10$

$4 \times 6 = 24 \quad 4 + 6 = 10,\ -4 + 6 = 2,\ 4 + (-6) = -2$

$1 \times 24 = 24 \quad 1 + 24 = 25,\ -1 + 24 = 23,\ 1 + (-24) = -23$

$3 \times 8 = 24 \quad 3 + 8 = 11, -3 + 8 = 5,\ 3 + (-8) = -5$

This is the correct product/sum pair

We then re-write the quadratic as

$2x^2 - 8x + 3x - 12$

Factorising each half separately gives

$2x(x - 4) + 3(x - 4)$

Taking out the common factor of the bracket then gives

$(x - 4)(2x + 3)$

Solving Quadratics by factorisation

You must be able to factorise quadratics in order to solve quadratic equations using this method.

Example 1

Solve $x^2 + 6x + 5 = 0$

This factorises into $(x + 5)(x + 1) = 0$

Each bracket needs to equal 0

$x + 5 = 0$ or $x + 1 = 0$

x = -5 or **x = -1**

Example 2

Solve $x^2 + 3x - 10 = 0$

This factorises into $(x + 5)(x - 2) = 0$

$x + 5 = 0$ or $x - 2 = 0$

x = -5 or **x = 2**

Example 3

Solve $x^2 - 6x + 9 = 0$

This factorises into $(x - 3)(x - 3) = 0$

This equation has repeated roots

$(x - 3)^2 = 0$

This means there is only one solution, **x = 3**

Further useful information

Check first that you can expand double brackets (using any appropriate method, such as using a grid or 'FOIL') e.g.

1. $(x + 1)(x - 6) = x^2 - 6x + x - 6$
   $= x^2 - 5x - 6$
2. $(2x - 3)(x + 4) = 2x^2 + 8x - 3x - 12$
   $= 2x^2 + 5x - 12$

Avoid being caught out!

- Sometimes a quadratic expression doesn't require double brackets e.g. $2x^2 - 7x = x(2x - 7)$
- Sometimes you can start by taking out a common factor e.g. $2x^2 - 72 = 2(x^2 - 36) = 2(x + 6)(x - 6)$

Online clips

U178, U858, U228, U960, U963

The quadratic formula

Component Knowledge

- Use the quadratic formula to find the solutions of any quadratic equation (if they exist)
- Calculate the discriminant of a quadratic and thus decide whether it has any roots

Key Vocabulary

Quadratic	An expression in which the highest power of x is 2, e.g. $x^2 + x - 1$ or $9 - x^2$
Coefficient	The number factor in an algebraic term, multiplied with variables (e.g. 4 in $4x$)
Discriminant	An expression involving the coefficients of a quadratic. It determines whether the quadratic has two roots, one or none.

The quadratic formula

The roots of the quadratic expression $ax^2 + bx + c$, that is the solutions of

$$ax^2 + bx + c = 0$$

are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Normally a calculator is used to find the roots using the quadratic formula

The notation $\pm$ is shorthand for two different formulas:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + 5x + 2 = 0$$

$a = +1$ $\quad b = +5$ $\quad c = +2$

$$x = \frac{-(+5) \pm \sqrt{(+5)^2 - (4 \times (+1) \times (+2))}}{2 \times (1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$x = \frac{-5 + \sqrt{17}}{2} \qquad x = \frac{-5 - \sqrt{17}}{2}$$

$x = -0.4$ $\qquad x = -4.6$

$$x^2 + 3x - 2 = 0$$

$a = +1$ $\quad b = +3$ $\quad c = -2$

$$x = \frac{-(+3) \pm \sqrt{(+3)^2 - (4 \times (+1) \times (-2))}}{2 \times (1)}$$

$$x = \frac{-3 \pm \sqrt{9 - (-8)}}{2}$$

$$x = \frac{-3 + \sqrt{17}}{2} \qquad x = \frac{-3 - \sqrt{17}}{2}$$

$x = 0.6$ $\qquad x = -3.6$

$$2x^2 - 5x + 1 = 0$$

$a = +2 \quad b = -5 \quad c = +1$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - (4 \times (+2) \times (+1))}}{2 \times (+2)}$$

$$x = \frac{5 \pm \sqrt{25 - 8}}{4}$$

$$x = \frac{5 + \sqrt{17}}{4} \qquad x = \frac{5 - \sqrt{17}}{4}$$

$x = 2.3 \qquad x = 0.2$

$$x^2 - 7x - 2 = 0$$

$a = +1 \quad b = -7 \quad c = -2$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - (4 \times (+1) \times (-2))}}{2 \times (1)}$$

$$x = \frac{7 \pm \sqrt{49 - (-8)}}{2}$$

$$x = \frac{7 + \sqrt{57}}{2} \qquad x = \frac{7 - \sqrt{57}}{2}$$

$x = 7.3 \qquad x = -0.3$

The *discriminant*

The expression $b^2 - 4ac$ which appears as the argument of the square root in the quadratic formula is called the *discriminant* of the quadratic. It is so called because it can be used to discriminate between three possible cases:

- If the discriminant is positive, $b^2 - 4ac > 0$, there are two roots, corresponding to the two operations indicated by $\pm$ in the formula. All the examples above fall into this category.
- If the discriminant is 0 then there is a single root, since $\sqrt{0} = 0$ and adding and subtracting 0 yield the same result. For example, $x^2 - 4x + 4 = 0$ has as solution only $x = 2$, because $b^2 - 4ac = 4^2 - 4 \times 1 \times 4 = 0$. (This can also be deduced from the fact that there is the factorisation $(x-2)^2 = 0$)
- If the discriminant is negative, $b^2 - 4ac < 0$, there are **no roots**, since a negative number does not have a square root. For example, there is no value of x such that $x^2 + 9x + 12 = 0$. (This can also be seen by *completing the square*: $(x+3)^2 + 3 = 0$)

  Geometrically, a negative discriminant means that the graph of the quadratic does not have any x-intercepts.

Online clips

U665

Completing the Square

Component Knowledge

- To be able to complete the square on quadratic equations
- To be able to apply completing the square in order to solve equations and find the turning point.

Key Vocabulary

Complete the Square	Re-writing an expression so it becomes a complete square and can be solved.
Quadratic	An equation or expression involving powers of 2.
Solve	Finding the values that make the equation true
Factorise	To break an expression down into its factors
Expand	Multiply terms inside a bracket
Term	Letter, symbols or numbers used in algebra
Root	The point or points at which a line crosses the x axis
Turning Point	A point where the direction of something changes

Completing the Square Method

Write $x^2 + 4x + 5$ in the form $(x + a)^2 + b$

$$\left(x + \frac{4}{2}\right)^2 = (x+2)^2$$

Step 1: Find a by dividing the number in front of the x term by 2 to make a perfect square

Step 2: Expand the new bracket and compare to the original expression

$$x^2 + 4x + 4$$

Step 3: Look at what needs adding to get back to the original expression (+1)

$$x^2 + 4x + 5$$

Step 4: Finish completing the square by writing in the form $(x + a)^2 + b$

$$(x+2)^2 + 1$$

So, a = 2 and b = 1

Solving by Completing the Square

Solve $x^2 - 8x + 15 = 0$

$$\left(x - \frac{8}{2}\right)^2 = (x-4)^2$$

$$x^2 - 8x + 16$$

$$x^2 - 8x + 15$$

(-1)

$$(x-4)^2 - 1 = 0$$

+1 | +1

$$(x-4)^2 = 1$$

$\sqrt{\ }$ | $\sqrt{\ }$

$$x - 4 = \pm 1$$

+4 | +4

$$x = 5 \text{ or } 3$$

Using Completing the Square to Find a Turning Point

Find the turning point of
$x^2 + 4x + 6 = 0$

$$\left(x + \frac{4}{2}\right)^2 = (x+2)^2$$

$$x^2 + 4x + 4$$

$$x^2 + 4x + 6$$

+2

$$(x+2)^2 + 2$$

This value gives the x co-ordinate but the sign is changed

This value gives the y co-ordinate

$$x = -2$$

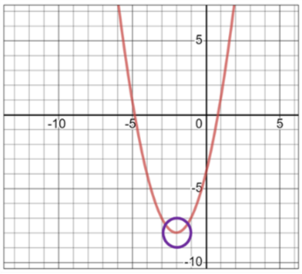
$$y = 2$$

$$\textit{Turning point} = (-2, 2)$$

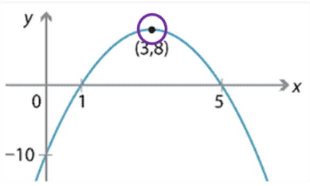
A turning point is the point where the graph has "turned around"

Turning points can be minimum or maximum points depending on the if the x^2 coefficient was positive or negative

We can find the turning point of a quadratic equation by completing the square



Minimum



Maximum

Sketching Quadratic Graphs

Sketch the graph of $x^2 + 2x - 3 = 0$

Step 1 – Factorise and solve or solve using quadratic equation

$(x + 3)(x - 1) = 0$

$x = -3$ and $x = 1$

Step 2 – Find y intercept

$y = x^2 + 2x - 3$

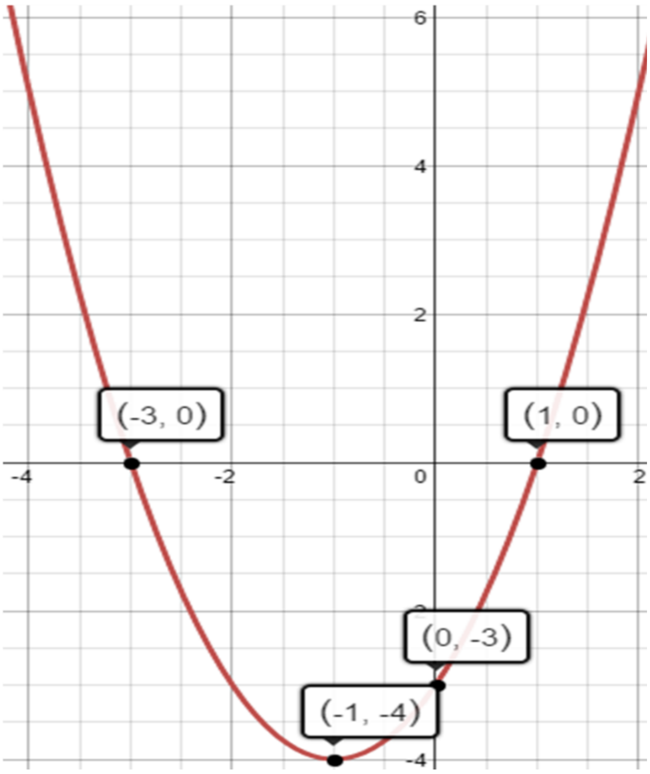
Step 3 – Complete the square

$y = (x + 1)^2 – 4$

Step 4 – Turning point using the completed square

(-1 , -4)

The Turning Point is either a **Maximum** value or a **Minimum** value.
If a quadratic is positive (U Shape) the turning point will be a **Minimum**.
If it is a negative (n Shape) it will be a **Maximum**



Online clips

U397, U589

Sketch quadratic graphs

Component Knowledge

To use Factorising a quadratic and completing the square to:

- Identify the roots
- Identify the y-intercept
- Identify the turning point.

Key Vocabulary

Quadratic	An equation where the highest power of a variable (usually x) is 2, e.g. it contains an x^2power but not an x^3 or higher. Written in the form $ax^2 + bx + c$
Roots	The values of x in a quadratic equation which give a value of y = 0. On a graph, this is where it crosses the x-axis.
Intercept	Where the graph crosses the y axis. The "c" part of the equation gives you the y part of the co-ordinate. The x part is always zero.
Turning point	A point where the graph changes from sloping downwards to upwards, or vice versa. Always written as a set of co-ordinates.
Parabola	A symmetrical, curved, U-shaped graph
Symmetry	A vertical line that divides the parabola into two congruent halves, through the turning point

Sketch quadratic graphs

Sketch the graph of $x^2 + 2x - 3 = 0$

Step 1 – Factorise and solve or solve using quadratic equation **$(x + 3)(x - 1) = 0$**

$x = -3$ and $x = 1$

Step 2 – Find y intercept

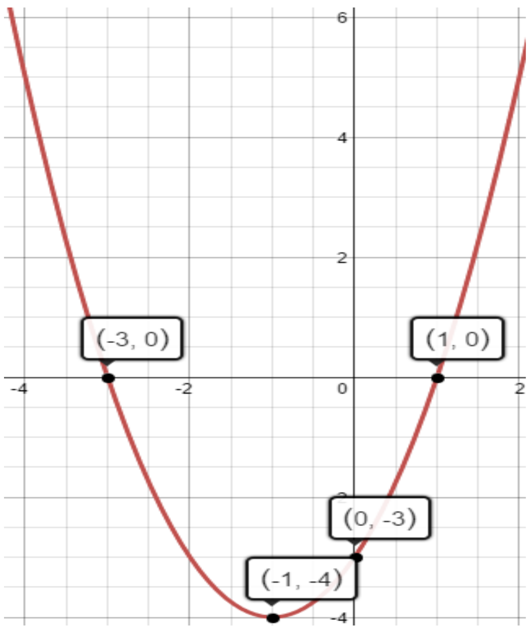
$y = x^2 + 2x - 3$

Step 3 – Complete the square

$y = (x + 1)^2 – 4$

Step 4 – Turning point using the completed square

(-1 , -4)



The Turning Point is either a **Maximum** value or a **Minimum** value.
If a quadratic is positive (U Shape) the turning point will be a **Minimum**.
If it is a negative (n Shape) it will be a **Maximum**

To find the coordinate of the turning point use the opposite sign to the value inside the bracket for the x value, the y value is the number added or subtracted at the end.

To find the **y intercept**, substitute x = 0 into the equation. This gives the y value of the coordinate; the x value of the coordinate will be 0. The y value will be the same as the constant term (the number value in the equation).

Online clips

U769, U989, U667

Plotting cubic graphs

Component Knowledge

- Complete a table of values for a cubic equation
- Plot a cubic graph
- Use a cubic graph to find approximate solutions to an equation

Key Vocabulary

Cubic	A polynomial which has an x^3 term as the highest power of x
Turning Point	Where a graph changes direction, either a maximum or a minimum point
Coordinate	A set of values that show an exact position
Approximate	A result that is not exact, but close enough to be used

Key Concepts

A **cubic graph** is a graphical representation of a cubic function.

A cubic is a polynomial which has an $\mathbf{x^3}$ term as the highest power of x.

Some cubic graphs have **two turning points** – a minimum and a maximum point.

A cubic graph with two turning points can touch or cross the x axis between one and three times.

How to plot a cubic graph

1) Complete the table of values
2) Plot the coordinates
3) Draw a smooth curve through the points

Using a cubic graph

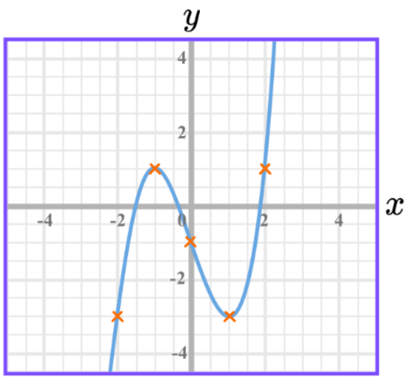
You can use a cubic graph to solve an equation by

1) Finding the given value on the y axis
2) Drawing a straight horizontal line across the curve
3) Drawing a straight vertical line from the curve to the x axis
4) Reading the values on the x axis

Example 1

Draw the curve of the equation $y = x^3 - 3x - 1$ for $-2 \leq x \leq 2$

X	-2	-1	0	1	2
y	-3	1	-1	-3	1

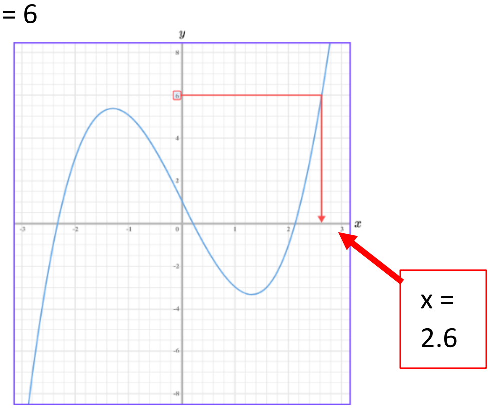


To complete the table of values, substitute each x value into the equation to calculate the y value.

Example 2

Use the graph of $y = x^3 - 5x + 1$ to find an approximate solution to the following equation

$x^3 - 5x + 1 = 6$



Online clips

U980

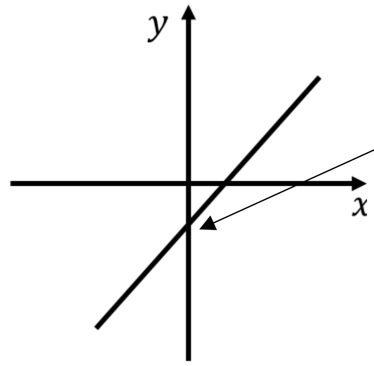
Types of graphs

Component Knowledge

- Identify the graph and associated equation of linear, quadratic, cubic, reciprocal, and exponential graphs.
- Recognise features of graphs (such as positive or negative gradient, intercepts, and asymptotes) and associate them with properties of the defining equation.

Key Vocabulary

Linear	A graph the equation of which has no second or higher powers of x, e.g. $y = 5x - 4$
Quadratic	A graph the equation of which has no powers of x higher than 2, e.g. $y = 2x^2 - x + 5$
Cubic	A graph the equation of which has no powers of x higher than 3, e.g. $y = x^3 - x^2 - 1$
Reciprocal	A graph in the equation of which has x in a denominator, e.g. $y = \frac{5}{x}$
Exponential	A graph the equation of which has x in a power, e.g. $y = 3 + 2^x$
Asymptote	A line that is approached by a graph (indefinitely closely) but not intersected
Intercept	The point where a graph meets one of the coordinate axes

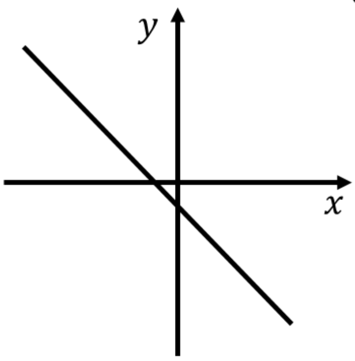


Negative $y-intercept$ means the **constant** term is negative, e.g. $y = 2x - 4$

Linear graph with positive *gradient* (upward slope) – positive coefficient of x, e.g.

$y = 6x$

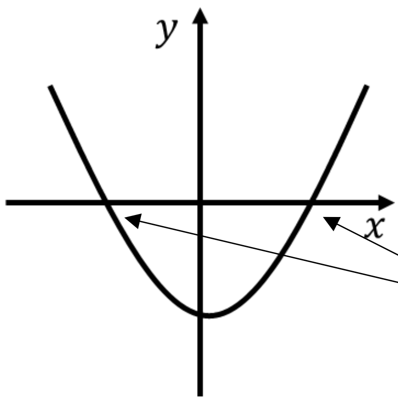
$y = 4 + x$



Linear graph with negative *gradient* (downward slope) – negative coefficient of x, e.g.

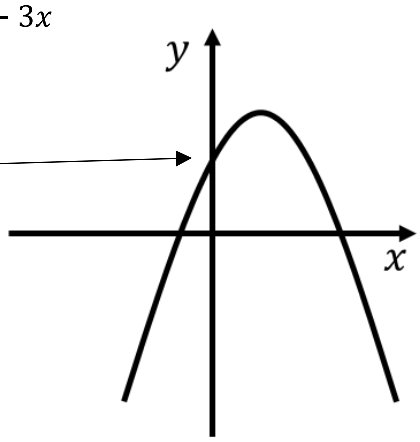
$y = -x$

$y = 10 - 3x$



Positive y-intercept means the **constant** term is positive, e.g. $y = -2x^2 + 1$

The x-intercepts are the **roots** of the quadratic



Quadratic graph with **minimum** *turning point* has **positive** coefficient of x^2, e.g.

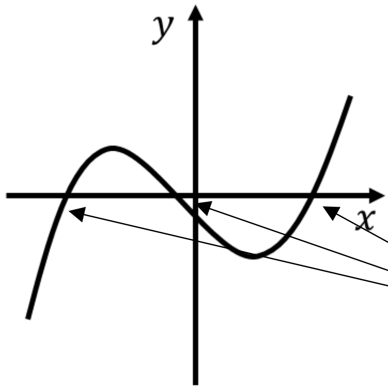
$y = x^2 - 6$

$y = 3x^2 - 5x + 2$

Quadratic graph with **maximum** *turning point* has **negative** coefficient of x^2, e.g.

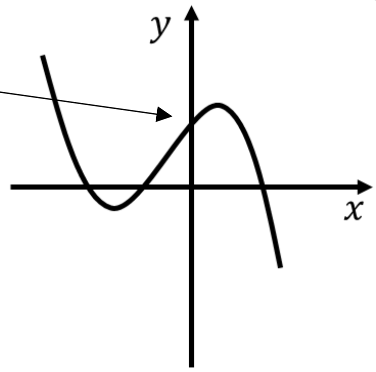
$y = 9 + x - x^2$

$y = -8x^2$



Positive y-intercept means the **constant** term is positive, e.g. $y = x - x^3 + 4$

The x-intercepts are the **roots** of the cubic



Cubic graph with **positive** coefficient of x^3, e.g.

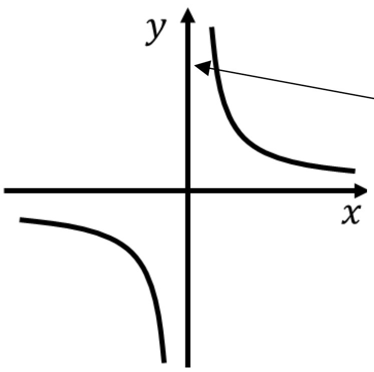
$y = x^3 - x^2 + 7$

$y = 2x^3 + x^2 - x$

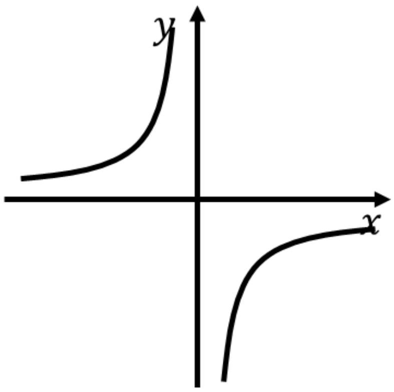
Cubic graph with **negative** coefficient of x^3, e.g.

$y = x - x^3$

$y = x^2 - 9 - 5x^3$



Reciprocal graphs have *asymptotes* (The graph **converges** towards the asymptote) In this case the graph will never meet the y axis.

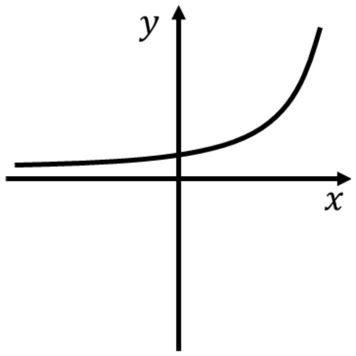


Reciprocal graph with **positive** coefficient of x, e.g.

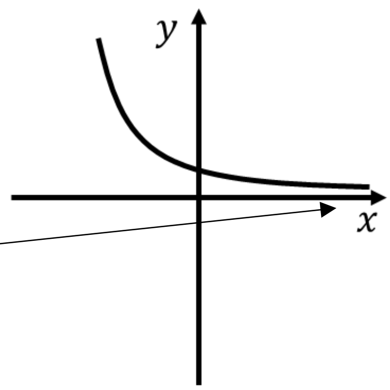
$y = \frac{1}{x}$ $\quad$ $y = \frac{5}{x^2}$

Reciprocal graph with **positive** coefficient of x, e.g.

$y = -\frac{6}{x}$ $\quad$ $y = -\frac{1}{2+x}$



Exponential graphs have *asymptotes* (The graph **converges** towards the asymptote)



Exponential graph with **positive** coefficient of x (and positive base > 1), e.g.

$y = 2^x$

$y = 5^x + 2$

Exponential graph with **negative** coefficient of x (and positive base> 1), e.g.

$y = 3^{-x}$

$y = 6^{10-2x}$

Online clips U593, U487, U980, U669, U667

Approximate Solutions to Equations Using Graphs

Component Knowledge

- Be able to draw straight line graphs.
- Be able to approximate solutions from graphs.

Key Vocabulary

Graph	A diagram showing the relationship between two quantities
Equation	A mathematical statement showing that two expressions are equal
Approximate	Close to an actual value but may not be completely accurate
Solution	Values for which an equation is true
Linear	Increasing or decreasing at a constant rate to form a straight line
Co-Ordinates	A pair of numbers used to describe the position of a point
Axis/Axes	A fixed reference line on a grid to help show the position of co-ordinates
Plot	To represent the relationship between two quantities graphically
Simultaneous	When two or more things occur at the same time
Intersect	When two or more things pass through each other

Equations describe a relationship between two values. We can plot this relationship on a graph to help us visualise the relationship more easily. By doing this we can also approximate solutions for given equations.

The equations of all straight lines can be written in the form:

$$y = mx + c$$

Gradient – The number in front of the x. This tells us how steep the line is.

Intercept – The number on its own. Shows where the line cuts the y axis.

Plotting Linear Graphs

Draw the graph of: $y = 2x + 1$

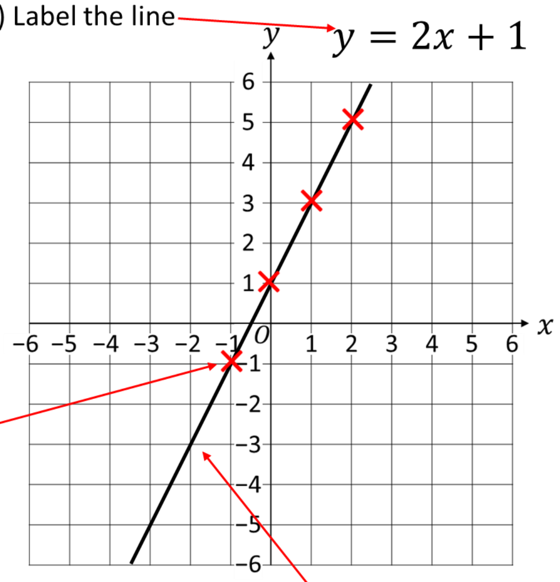
x	-1	0	1	2	3
y	-1	1	3	5	7

$(-1,-1)$ $(0,1)$ $(1,3)$ $(2,5)$ $(3,7)$

$E.g. (2 \times 2) + 1 = 5$

1) Complete the table of values by substituting x values.

2) Plot each pair of values as coordinates.



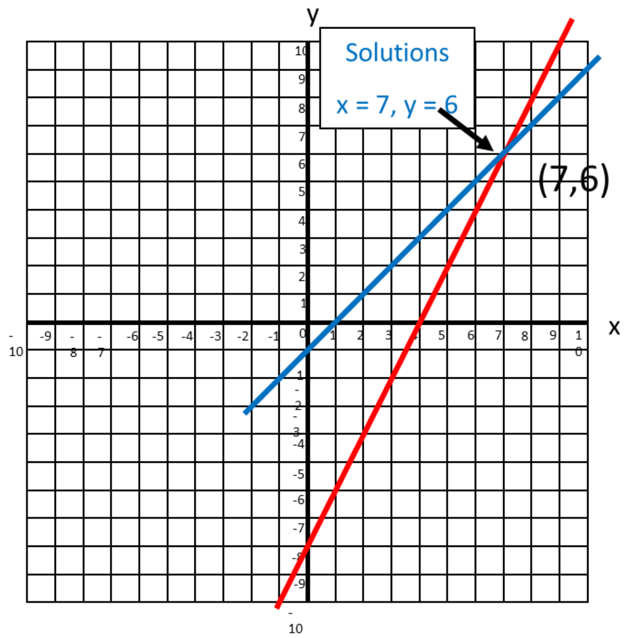
3) Join the points to make a line.

4) Label the line

Solving Simultaneous Equations When the Graph is Given

Solve the simultaneous equations

2x – y = 8 and x – y = 1



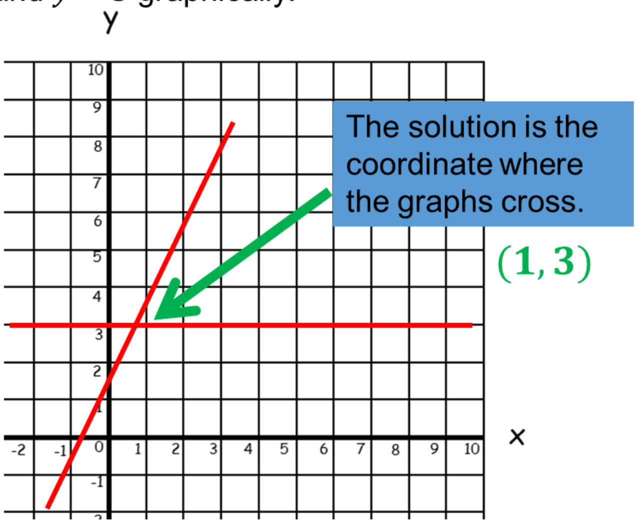
The solution to the simultaneous equation is where the two lines intersect.

There will always be two solutions one for x and one for y.

So $x = 7$ and $y = 6$.

Plotting and Solving Graphically

Solve the simultaneous equations $y = 2x + 1$ and $y = 3$ graphically:



First sketch $y = 3$.

Draw a table of values to sketch $y = 2x + 1$ and plot the line

x	0	1	1
y	1	3	5

So $x = 1$ and $y = 3$.

Sometimes we are asked to show the solution graphically but not given the graphs.

In these cases we must first plot the graphs as shown previously.

Online clips

U836

Simultaneous linear equations

Component Knowledge

- Solving simultaneous linear equations with a balanced variable by elimination
- Solving simultaneous linear equations where balancing a variable is required
- Form and solve simultaneous equations.

Key Vocabulary

Simultaneous equations	Two or more equations that are two be solved (if possible) by using the *same* value for each variable
Coefficient	The number factor in an algebraic term, multiplied with variables (e.g. 4 in $4x$)
Balancing variables	Equating the coefficients of like terms in different equations by multiplying with suitable factors
Eliminating variables	Reducing the term containing a particular variable in an equation to 0 by subtracting/adding another equation with the same/opposite term
Substitution	Assigning a value to a variable (e.g. substituting $y = 8$ in $6y$ gives 48)

Solving simultaneous equations – no balancing needed

In the first example, because the two equations have **equal** terms in x – both are $3x$ – *subtracting* the equations (remember to subtract both sides) *eliminates* the x term. The resulting equation has only one unknown, y, and can be solved.

$$3x + 4y = 29$$
$$-\quad 3x + 2y = 19$$
$$2y = 10$$
$$y = 5$$

Substitute y into either equation to find x.

$$3x + (2 \times 5) = 19$$
$$3x + 10 = 19$$
$$3x = 9$$
$$x = 3$$

Here the value found for y is **substituted** into the second equation to obtain an equation in terms of x. The first equation could have been used too.

Whichever equation is used for substitution, it is good practice to check the pair of values found in the other equation too, to ensure no mistakes have been made:

$$3 \times 3 + 2 \times 5 = 19$$

In the second example, because the two equations have **opposite** terms in y – one is $2y$ and the other $-2y$ – *adding* the equations eliminates the y term.

$$3x + 2y = 16$$
$$+\quad 2x - 2y = 4$$
$$5x = 20$$
$$x = 4$$

Substitute x back in to find y.

$$(2 \times 4) - 2y = 4$$
$$8 - 2y = 4$$
$$8 = 4 + 2y$$
$$4 = 2y$$
$$2 = y$$

Forming simultaneous equations – balancing a variable

$$2x + 8y = 32$$
$$x + 3y = 13 \quad (\times 2)$$
$$\star \quad 2x + 6y = 26$$
$$2y = 6$$
$$y = 3$$

$$2x + 6(3) = 26$$
$$2x + 18 = 26$$
$$2x = 8$$
$$x = 4$$

Here neither the x nor the y terms are already balanced. But the x terms can be balanced by multiplying the second equation by 2. (Remember to **multiply both sides** by the factor.)
The modified second equation can then be subtracted from the first, and the subsequent steps are as before.

$$5x + 4y = 19 \quad (\times 3)$$
$$2x - 3y = 3 \quad (\times 4)$$
$$8x - 12y = 12$$
$$15x + 12y = 57$$
$$23x = 69$$

$$x = 3$$

$$15 + 4y = 19$$
$$4y = 4$$
$$y = 1$$

In this example the y terms can be balanced by multiplying the first equation by 3 and the second by 4, since 12 is the lowest common multiple of the starting coefficients. (Alternatively, we can balance the x terms. What factors would be needed in that case?)
The modified equations are then added – since the y terms have opposite signs – and the following steps are as before.

Forming simultaneous equations to solve a problem

Barry buys 200 pieces of stationery for £76.

Of the 200 pieces of stationery, x of them are rulers that cost 50p each and y of them are pens that cost 20p each.

Find how many rulers and pens Barry buys.

The information in the question can be written as the simultaneous equations

$x + y = 200$

$50x + 20y = 7600$ (amounts are written in pence)

Multiply the first equation by 50 to give $50x + 50y = 10000$. The x terms are now balanced, and subtracting the second equation gives $30y = 2400$.

Therefore $y = 80$, and using the first equation $x = 120$.

Online clips

U760

Solving quadratic Simultaneous equations

Component Knowledge

- To be able to solve a simultaneous equation made up of a linear equation and a quadratic equation using algebra.
- To be able to solve a simultaneous equation made up of a linear equation and a quadratic equation using graphs.

Key Vocabulary

Linear equation	An equation between two variables that gives a straight line when plotted on a graph
Quadratic equation	Quadratic algebraic equations are equations that contain terms up to x^2
Simultaneous equations	Simultaneous equations require algebraic skills to find the values of letters within two or more equations
Intersection	The point where two lines meet

One linear equation and one quadratic equation

$$x^2 + y^2 = 17$$

$$y = x - 3$$

Substitute y = x – 3 into the quadratic equation

$$x^2 + (x - 3)^2 = 17$$

$$x^2 + x^2 - 6x + 9 = 17$$

Make the equation equal to 0

$$x^2 + x^2 - 6x + 9 - 17 = 0$$

$$2x^2 - 6x - 8 = 0$$

Solve by factorising or using the quadratic formula

$2x^2 - 6x - 8 = 0$ factorises to (2x + 2) (x-4)

$2x + 2 = 0$ gives the solution x = -1

$x - 4 = 0$ gives the solution x = 4

Substitute the x values into the linear equation to find the corresponding y values

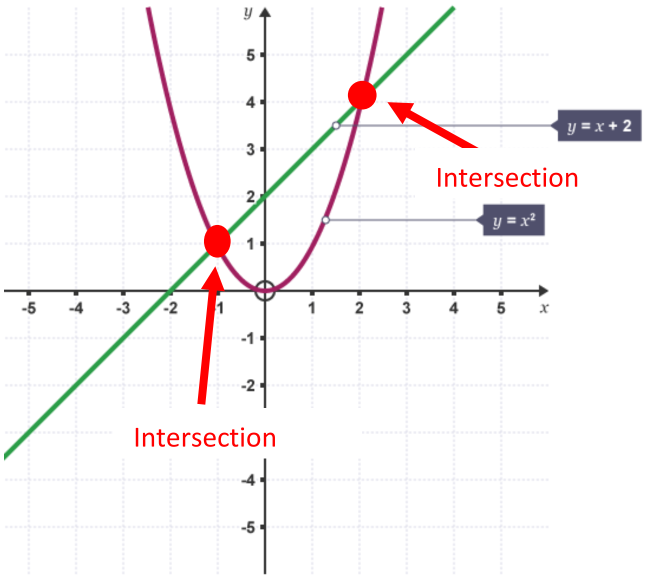
When x = 4 y = 4 – 1 = 1

When x = -1 y = -1 -3 = -4

One linear graph and one quadratic graph

Simultaneous equations that contain a quadratic and equation can also be solved graphically. As with solving algebraically, there will usually be two pairs of solutions.

Plot the graphs on the axes and look for the points of intersection



The two points of intersection are at (2, 4) and (-1, 1) so x=2 and y=4, and x=−1 and y=1.

Online clips U547, U875, U269

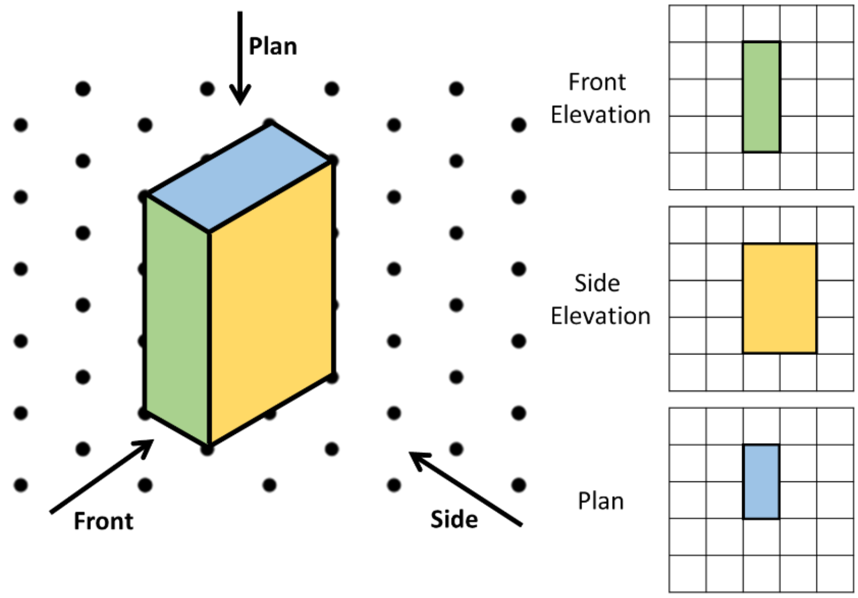
Plans and elevations

Component Knowledge

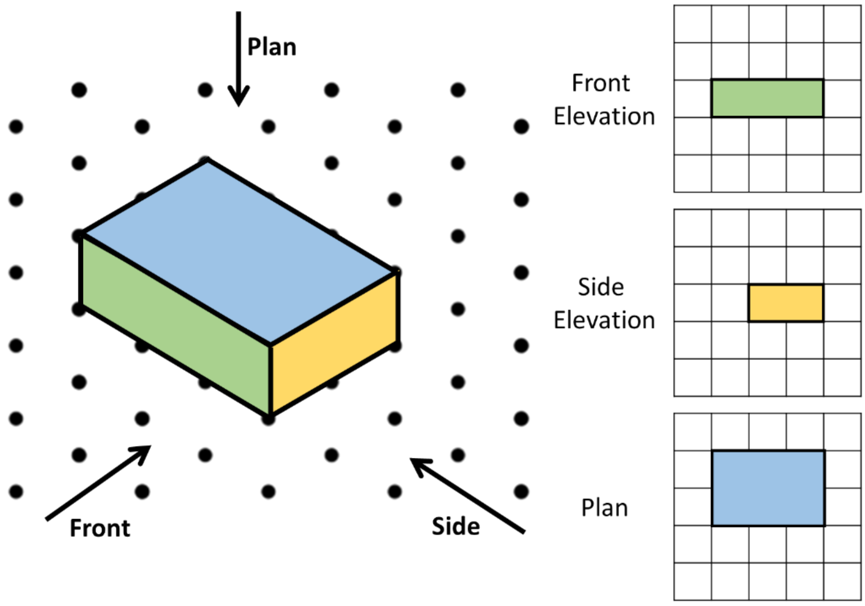
- Draw the plan of an oriented 3-dimensional shape
- Draw the front elevation (direction specified) of a 3-dimensional shape
- Draw the side elevation (direction specified) of a 3-dimensional shape

Key Vocabulary

Plan	The view of an oriented 3-dimensional shape from above
Front elevation	The view of an oriented 3-dimensional shape from a specified front direction
Side elevation	The view of an oriented 3-dimensional shape from the side



Ensure that the dimensions of the plan and the elevation are consistent with the lengths of the 3-dimensional shape.

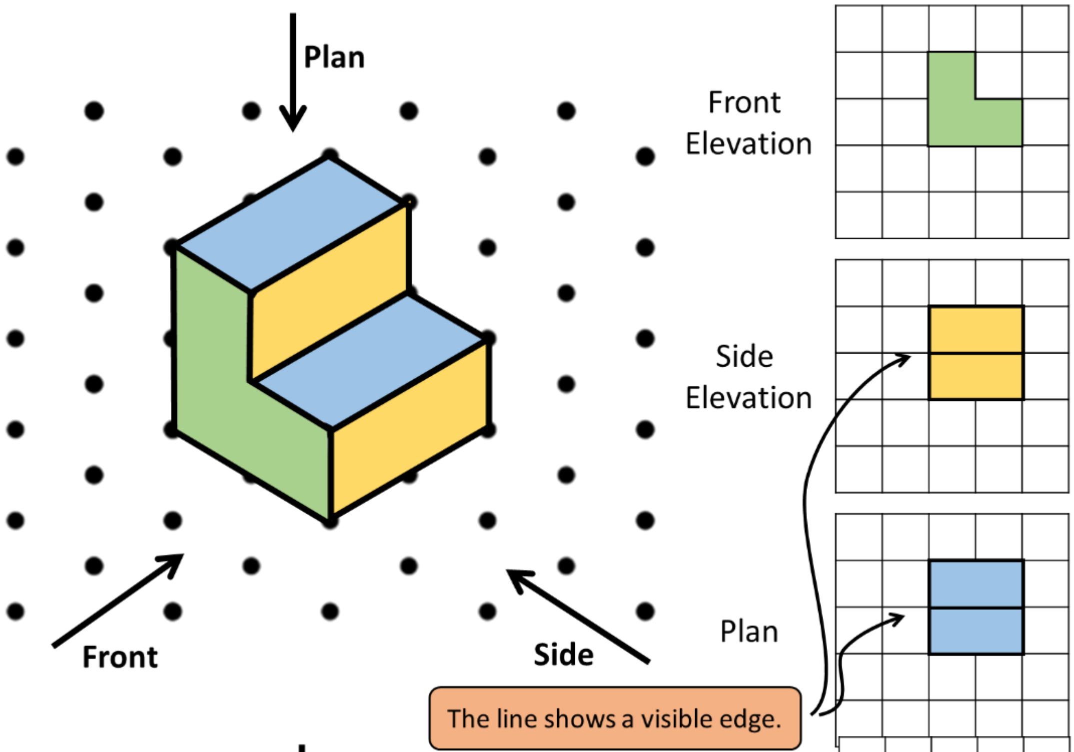


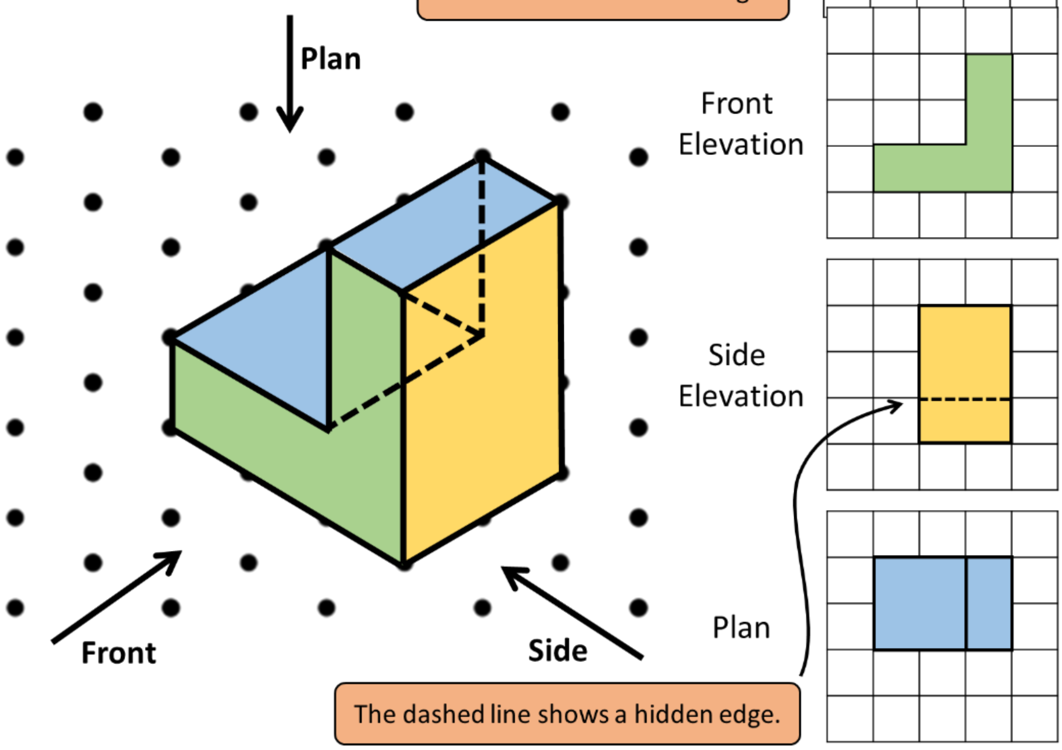
If only the front direction is specified, both the left and right-side view are acceptable as the side elevation.

(They are either the same, or mirror images)

Showing edges in plans and elevations

This provides more information about the shapes and makes it easier to identify the direction from which the plan and elevation are drawn.





Online clips

M229