Congruency

Component Knowledge

- Know what congruency is and how to identify whether shapes are congruent
- Recognise congruent triangles
- Know the rules for congruency

Key Vocabulary

Congruent	The same shape and size
Triangle	A 3 sided flat shape with straight sides
Hypotenuse	The side opposite the right angle in a right angles triangle
Right angle	An angle which is equal to 90°
Identical	Exactly the same
Side	One of the line segments that make a flat 2D shape

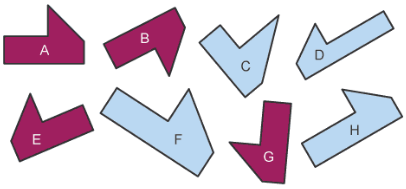
Key Concepts

Shapes are **congruent** if they are **identical** – same shape and same size.

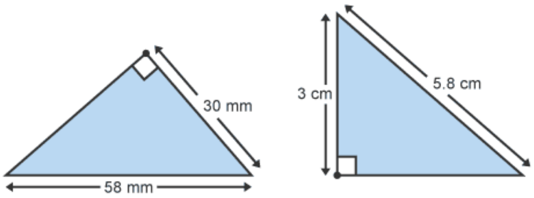
Shapes can be rotated or reflected but still be congruent.

Examples

Which shapes are congruent?



Shapes A, B, E and G are congruent as they are identical in size and shape.



These are congruent, they both have a right angle, the same hypotenuse and another side the same

Triangles

There are four ways of proving that two triangles are congruent:

1) **SSS** (Side, Side, Side)
   a. All 3 sides are the same in both triangles
2) **RHS** (Right angle, Hypotenuse, Side)
   a. Both triangles have a right angle, the same hypotenuse and one other side the same
3) **SAS** (Side, Angle, Side)
   a. Two sides with the angle in between them are the same in both triangles
4) **ASA** (Angle, side, Angle) or **AAS**
   a. One side and two angles are the same in both triangles

Misconceptions

Proving all 3 angles are the same is **not** proving they are congruent, as one could be an enlargement of the other.

Angle, Side, Side is **not** a proof for congruency as the angle needs to be contained between the two sides.

Online clips

U790, U112, U866

Pythagoras Theorem

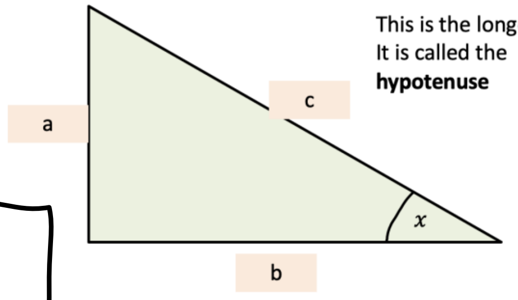
Component Knowledge

- Identify the hypotenuse in a right-angled triangle.
- Use substitution in formula.
- Solve an equation by rearranging

Key Vocabulary

Hypotenuse	The longest side in a right-angled triangle
Opposite	The side facing the given angle in a right-angled triangle
Adjacent	The side next to the given angle in a right-angled triangle
Square number	The result when you multiply a number by itself.

Key Properties:



This is the longest side. It is called the **hypotenuse**

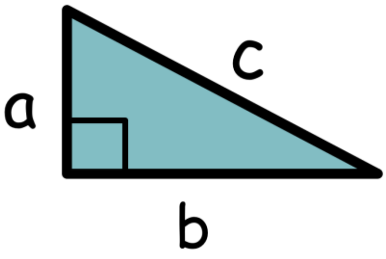
This is the **opposite** side to "angle x"

This is the **adjacent** side to "angle x"

Note – the hypotenuse is always the side facing the right angle

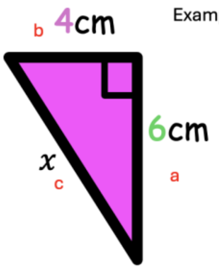
The formula:

$$a^2 + b^2 = c^2$$



Using Pythagoras Theorem to find missing sides

Finding the hypotenuse:



Example: Find x to 1dp

- Step 1 – Label all the sides
- Step 2 – substitute values into the formula

$a^2 + b^2 = c^2$

$6^2 + 4^2 = c^2$

Solve the equation

$36 + 4 = c^2$

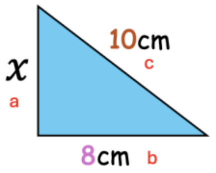
$40 = c^2$

$(\sqrt{\ })$ $(\sqrt{\ })$

$6.426 = c$

- Step 3 – round the number and don't forget your units
- x = 6.4cm (to 1dp)

Finding a shorter side:



Example: Find x to 1dp

- Step 1 – Label all the sides
- Step 2 – substitute values into the formula

$a^2 + b^2 = c^2$

$a^2 + 8^2 = 10^2$

Solve the equation

$a^2 + 64 = 100$

(-64) (-64)

$a^2 = 36$

$(\sqrt{\ })$ $(\sqrt{\ })$

$a = 6$

- Step 3 – round the number and don't forget your units
- x = 6cm

Online Clips

U385, U828

Trigonometry

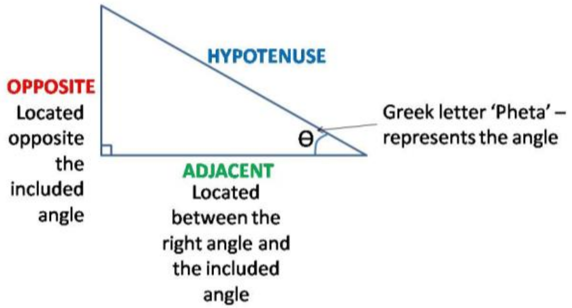
Component Knowledge

- Recall the three trigonometric ratios
- Correctly label the sides of a right-angled triangle with Opp, Adj, Hyp
- Identify the correct trigonometric ratios to use.
- Use the correct trigonometric ratio to find the missing side or angle.
- To identify exact values for key angles using trigonometric ratios.

Key Vocabulary

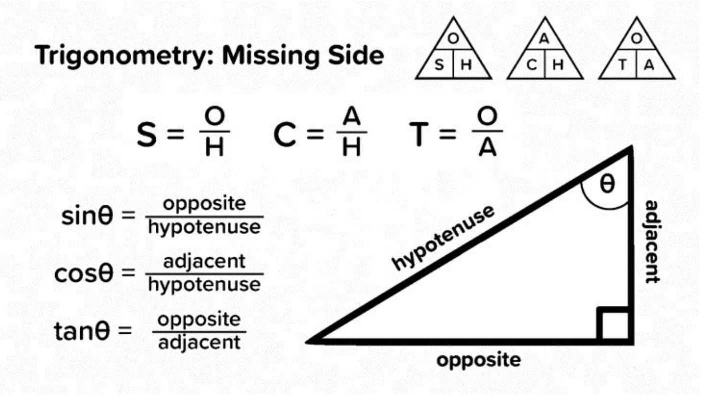
Trigonometry (trig)	Trigonometry helps us find angles and lengths in right-angled triangles.
Trigonometric ratios	There are three trigonometric ratios, depending on the position of the unknown sides and angles.
Hypotenuse side (Hyp)	The length of the longest side of a right-angled triangle.
Opposite side (Opp)	The length of the side opposite the given angle.
Adjacent side (Adj)	The length of the side next to the given angle and right-angle.
Sine ratio (sin)	Is used to describe the relationship between the given angle, Opposite side and Hypotenuse
Cosine ratio (cos)	Is used to describe the relationship between the given angle, Adjacent side and Hypotenuse
Tangent ratio (tan)	Ratio used to describe the relationship between the given angle, Opposite side and Adjacent side.

Labelling the sides



Hypotenuse will **always** be opposite the 90° angle. The Opposite and Adjacent will change depending on where the given angle (Θ) is on the diagram.

Trig ratios



$S = \frac{O}{H}$ $C = \frac{A}{H}$ $T = \frac{O}{A}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

Finding the missing side

- numerator

Remember to show your full calculator answer too, should you have a decimal, and then round to get your final answer.

To find the length of b

Step 1: Label each side
Step 2: Choose the correct formula
Since we know the hypotenuse and need to find the adjacent side, we can use cos
Step 3: Substitute the values into the formula

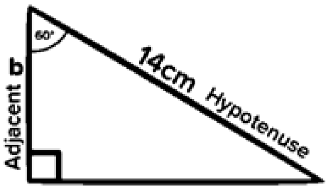
$\cos(60) = \frac{b}{14}$

Step 4: Rearrange to find b

$b = \cos(60) \times 14$

b = 7

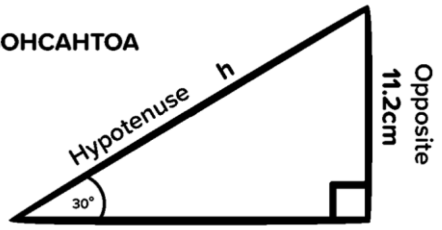
SOH CAH TOA



Finding the missing side- denominator

Q Find the length of h

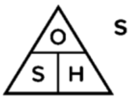
SOHCAHTOA



Step 1 Label each side

Step 2 Choose the correct formula

Since we know the opposite side and need to find the hypotenuse, we can use sin



$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

Step 3 Substitute the values into the formula

$\sin(30) = \frac{11.2}{h}$

Step 4 Rearrange to find h

$h = \frac{11.2}{\sin(30)}$

A 22.4cm

Here we have multiplied both sides by h and then divided both sides by sin (30)

Finding the missing side- angle

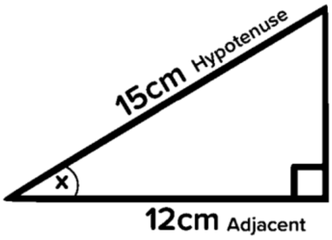
To type $\sin^{-1}$ into your calc, press SHIFT and then the SIN button.

To type $\cos^{-1}$ into your calc, press SHIFT and then the COS button.

To type $\tan^{-1}$ into your calc, press SHIFT and then the TAN button.

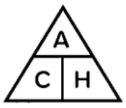
Q Calculate the value of the missing angle x

Give your answer to 1 decimal place.



Step 1 Label the triangle

Step 2 Choose the correct formula



$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

Step 3 Substitute and solve

$\cos(x) = \frac{12}{15}$

$x = \cos^{-1}\left(\frac{12}{15}\right)$

$= 36.8698...$

1 decimal place

A 36.9°

Exact trigonometric values- LEARN BY HEART!!

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$undefined$

Online clips

U605, U283, U545, U627

Angles on Parallel lines

Component Knowledge

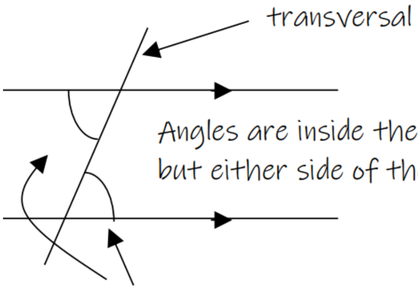
- Basic angle facts – such as angles on a straight line = $180°$
- Recognise that a traversal is a line which crosses a set of parallel lines
- To be able to find missing angles on parallel lines.

Key Vocabulary

Parallel lines	Lines are parallel if they are always the same distance apart (called "equidistant"), and will never meet
Supplementary	Two angles are supplementary when they add up to 180˚

Parallel Lines Angle Facts:

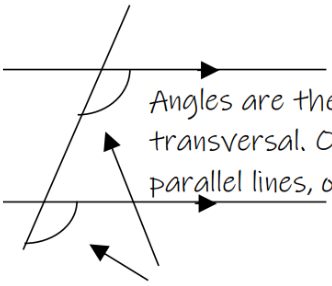
Alternate Angles



Angles are inside the parallel lines but either side of the transversal

Alternate angles are equal (Z shape)

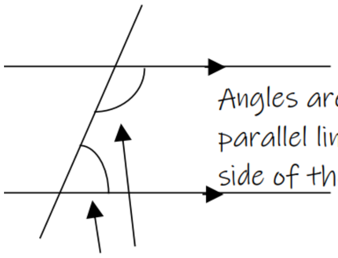
Corresponding angles



Angles are the same side of the transversal. One inside the parallel lines, one outside.

Corresponding angles are equal (F shape)

Co-Interior Angles

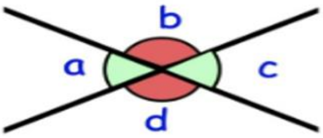


Angles are both inside the parallel lines and on the same side of the transversal.

Allied angles sum to 180° (C shape)

These are also called co-interior angles

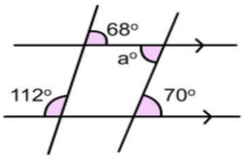
Vertically opposite angles



Vertically opposite angles are equal

a = c and b = d

Examples

	a = 70˚ Reason -Alternate Angles are equal

Important! ALWAYS state the angle and the reason.

Online Clips

U390, U730, U628, U732, U655, U826

Surds

Component Knowledge

- Simplify a surd
- Multiply/Divide surds
- Add/Subtract surds
- Rationalise a denominator

Key Vocabulary

Surd	A non-square number, a number that can only be written as a root, a non - terminating decimal without repetition
Irrational	A number that **cannot** be written as a fraction (or hence as an integer)
Rational	A number that **can** be written as a fraction (or hence as an integer)
Rationalise	Remove any terms with roots
Denominator	“bottom” of a fraction, indicates the “type” of fraction, what you are diving by
Radicand	The value whose root is taken, the number under a root
Square	The result of multiplying a value by itself

Surds:

$\sqrt{5}$ **is** a surd as $\sqrt{5} = 2.2360679774997$ … an **irrational number**, written as a square root, a decimal that “goes on” forever without a repeating pattern

$\sqrt{9}$ is **not** a surd as $\sqrt{9} = 3$ an **integer**

$\sqrt{\frac{1}{4}}$ is **not** a surd as $\sqrt{\frac{1}{4}} = \frac{1}{2}$ a **fraction**

$\sqrt{1.44}$ is **not** a surd as $\sqrt{1.44} = 1.2$ a **terminating** decimal and hence can be written as $\frac{12}{10} = \frac{6}{5}$

Simplifying Surds:

Simplify $\sqrt{48}$ list the factor pairs of 48

Identify the ***largest*** square number factor of 48

$= \sqrt{16 \times 3}$ Write the radicand as this factor pair

$= \sqrt{16} \times \sqrt{3}$ Separate into individual roots

$= 4 \times \sqrt{3}$ Evaluate any roots you can

$= 4\sqrt{3}$ Write in conventional notation, no × sign

Factor Pairs of 48

1×48

2×24

3×16

4×12

6×8

Multiplying/Dividing Surds:

$2\sqrt{6}\times\sqrt{15}$	put under a single root	$\sqrt{24}\div\sqrt{3}$
$2\sqrt{6\times 15}$	now ***simplify*** as earlier	$\sqrt{24\div 3}$
$2\sqrt{90}$	now ***simplify*** as earlier	$\sqrt{8}$
$2\sqrt{9\times 10}$	**largest** square factor	$\sqrt{4\times 2}$
$2\sqrt{9}\times\sqrt{10}$		$\sqrt{4}\times\sqrt{2}$
$2\times 3\times\sqrt{10}$		$2\times\sqrt{2}$
$6\sqrt{10}$		$2\sqrt{2}$

Multiplying/Dividing Surds:

In general: and

$\sqrt{a}\times\sqrt{b}$	$\sqrt{a}\div\sqrt{b}$
$\sqrt{a\times b}$	$\sqrt{a\div b}$
$\sqrt{ab}$	$\sqrt{\frac{a}{b}}$

WARNING:

This only works for **multiplying and dividing** not for addition and subtraction.

$$\sqrt{a}\pm\sqrt{b}\neq\sqrt{a\pm b}$$

Adding Subtracting Surds:

$4\sqrt{3}+6\sqrt{3}$ collect any surds with common radicands (may need to simplify first)

$10\sqrt{3}$

Surds can only be added or subtracted if they have a ***common*** (the same) ***radicand*** (number under the root)

Online clips: U338, U633, U872,

Surds –

Expanding Brackets

Component Knowledge

- Expand a single bracket with surds
- Expand a pair of double brackets with surds

Key Vocabulary

Surd	A non-square number, a number that can only be written as a root, a non - terminating decimal without repetition
Irrational	A number that **cannot** be written as a fraction (or hence as an integer)
Rational	A number that **can** be written as a fraction (or hence as an integer)
Rationalise	Remove any terms with roots
Denominator	“bottom” of a fraction, indicates the “type” of fraction, what you are diving by
Radicand	The value whose root is taken, the number under a root
Square	The result of multiplying a value by itself

Expand a single bracket with surds:

Expand $\sqrt{3}(5-\sqrt{6})$ multiply each term inside the bracket by $\sqrt{3}$

$5\sqrt{3}-\sqrt{3\times 6}$ write $\sqrt{3\times 6}$ as a single radicand

$5\sqrt{3}-\sqrt{18}$ simplify $\sqrt{18}$ as before

$5\sqrt{3}-3\sqrt{2}$

Expand a pair of double brackets with surds:

Expand $(5-\sqrt{2})(\sqrt{6}+1)$ multiply each term inside the 2nd bracket by 5,

multiply each term inside the 2nd bracket by $-\sqrt{2}$,

$5\sqrt{6}+5-\sqrt{2}\sqrt{6}-1\sqrt{2}$ manipulate

$5\sqrt{6}+5-\sqrt{12}-1\sqrt{2}$ simplify, as before, any surds you can

$5\sqrt{6}+5-\sqrt{4\times 3}-1\sqrt{2}$

$5\sqrt{6}+5-2\sqrt{3}-1\sqrt{2}$ check for any common radicands that could be added/subtracted

Online clips

U179, U768, U499

Surds –

Rationalising denominator

Component Knowledge

- Rationalise a single term denominator
- Rationalise a two term denominator

Key Vocabulary

Surd	A non-square number, a number that can only be written as a root, a non - terminating decimal without repetition
Irrational	A number that **cannot** be written as a fraction (or hence as an integer)
Rational	A number that **can** be written as a fraction (or hence as an integer)
Rationalise	Remove any terms with roots
Denominator	"bottom" of a fraction, indicates the "type" of fraction, what you are diving by
Radicand	The value whose root is taken, the number under a root
Square	The result of multiplying a value by itself

Rationalise single term denominator:

Rationalise the denominator of $\frac{2\sqrt{3}}{\sqrt{5}}$

the ***denominator*** cannot have a root in it

$\sqrt{5}$ can be rationalised by multiplying by $\sqrt{5}$, find an equivalent fraction by both parts of the fraction by $\sqrt{5}$

$$\frac{2\sqrt{3}}{\sqrt{5}} \begin{matrix} \times\sqrt{5} \\ \times\sqrt{5} \end{matrix}$$

$$\frac{2\sqrt{3}\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$$

$$\frac{2\sqrt{15}}{\sqrt{25}}$$

$$\frac{2\sqrt{15}}{5}$$

Rationalise two term denominator:

Rationalise the denominator of $\frac{7\sqrt{5}}{6+\sqrt{2}}$

the ***denominator*** cannot have a root in it

To leave no terms with $\sqrt{2}$ in the denominator we must by multiply by $(6-\sqrt{2})$, multiplying by $\sqrt{2}$ alone would leave $6\sqrt{2}+2$ in the denominator (*this is an application of a difference of 2 squares quadratic*)

$$\frac{7\sqrt{5}}{6+\sqrt{2}} \begin{matrix} \times(6-\sqrt{2}) \\ \times(6-\sqrt{2}) \end{matrix}$$

expand out

$$\frac{42\sqrt{5}-7\sqrt{5}\sqrt{2}}{36-6\sqrt{2}+6\sqrt{2}-\sqrt{2}\sqrt{2}}$$

simplify

$$\frac{42\sqrt{5}-7\sqrt{10}}{36-2}$$

simplify

$$\frac{42\sqrt{5}-7\sqrt{10}}{34}$$

Online clips

U633, U707, U281

Expand triple brackets

Component Knowledge

- Expand triple brackets (all positive terms)
- Expand triple brackets (both positive and negative terms)

Key Vocabulary

Expand	Multiplying expressions to remove the brackets
Brackets	Symbols used in pairs to group things together
Simplify	An expression is in its simplest form when it is easiest to use
Quadratic	Where the highest exponent of the variable (usually "x") is a square (^{2}). It will look something like x^2

Expand triple brackets (all positive)

To expand triple brackets, we multiply two brackets together and then multiply that expression by the final bracket. This is the same method as multiplying 3 numbers, e.g. 3 x 4 x 5 = (3 x 4) x 5 = 12 x 5 = 60.

Example

Expand and simplify

$$\underbrace{(x+1)(x+2)}(x+5)$$

$$x^2+2x+x+2$$

$$\mathbf{x^2+3x+2}$$

$$(x^2+3x+2)(x+5)$$

$$x^3+5x^2+3x^2+15x+2x+10$$

$$\mathbf{x^3+8x^2+17x+10}$$

Example

Expand and simplify $(x+4)(x+1)(x-2)$

So $(x+4)(x+1)(x-2) \equiv (x+4)(x^2-x-2)$

$\times$	x	$+1$
x	x^2	$+x$
$+4$	$+4x$	$+4$

$\equiv x^2+4x+x+4$

$\equiv x^2+5x+4$

$\times$	x^2	$+5x$	$+4$
x	x^3	$+5x^2$	$+4x$
-2	-$2x^2$	-$10x$	-8

$\equiv x^3+5x^2-2x^2+4x-10x-8$

$\underline{\equiv x^3+3x^2-6x-8}$

Expand triple brackets (both positive and negative)

Example

Expand and simplify

$$\underbrace{(x+1)(x+2)}(x-5)$$

$$\equiv x^2+2x+x+2$$

$$\equiv x^2+3x+2$$

$$(x^2+3x+2)(x-5)$$

$$\equiv x^3-5x^2+3x^2-15x+2x-10$$

$$\underline{\equiv x^3-2x^2-13x-10}$$

Example

Expand and simplify $(x+4)(x+1)(x-2)$

So $(x+4)(x+1)(x-2) \equiv (x^2+5x+4)(x-2)$

$\times$	x	$+1$
x	x^2	$+x$
$+4$	$+4x$	$+4$

$\equiv x^2+4x+x+4$

$\equiv x^2+5x+4$

$\times$	x^2	$+5x$	$+4$
x	x^3	$+5x^2$	$+4x$
-2	$-2x^2$	$-10x$	-8

$\equiv x^3+5x^2-2x^2+4x-10x-8$

$\underline{\equiv x^3+3x^2-6x-8}$

Online clip U606

3D Pythagoras' Theorem and Trigonometry

Component Knowledge

- To be able to calculate the length of a side using Pythagoras' Theorem in a 3D shape.
- To be able to calculate the length of a side using trigonometry in a 3D shape.
- To be able to calculate the size of an angle using trigonometry in a 3D shape.

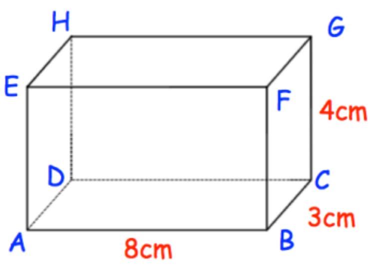
Key Vocabulary

Right angle	A 90° angle
Squared	A number that has been multiplied by itself has been squared
Square root	The value which when multiplied to itself gives the original number
Hypotenuse	The longest side of a right-angled triangle
Adjacent	The side next to the angle in trigonometry
Opposite	The side opposite the angle in trigonometry

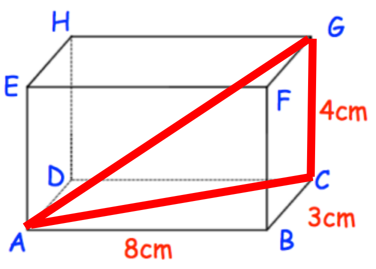
3D Pythagoras' Theorem

Find missing lengths by identifying right angled triangles. You will often have to find a missing length you are not asked for before finding the missing length you are asked for.

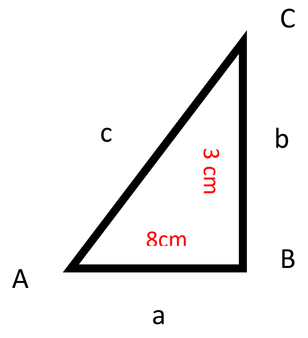
Worked Example. Calculate the length of AG.



1st step: Identify the 2D triangle within the 3D shape which includes the side that you need to find the length of.



2nd step: Find the length of the base of the triangle using Pythagoras' theorem. We can now find the length of the side AC.



$c^2 = a^2 + b^2$

$c^2 = 8^2 + 3^2$

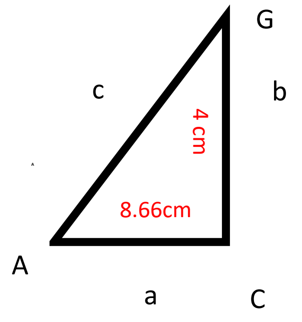
$c^2 = 64 + 9$

$c^2 = 75$

$c = \sqrt{75}$

$c = 8.66$ cm

3rd step: Using the length we have just calculated find the length of side in the question.



$c^2 = a^2 + b^2$

$c^2 = 8.66^2 + 4^2$

$c^2 = 75 + 16$

$c^2 = 91$

$c = \sqrt{91}$

$c = 9.5$ cm

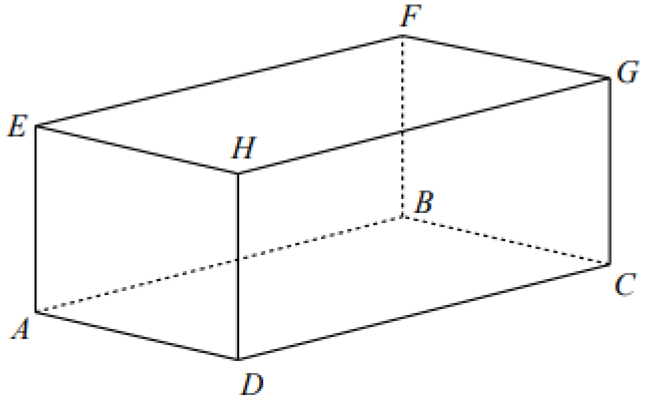
3D Trigonometry – calculating the length of a side

Worked Example.

The diagram shows a cuboid *ABCDEFGH*.

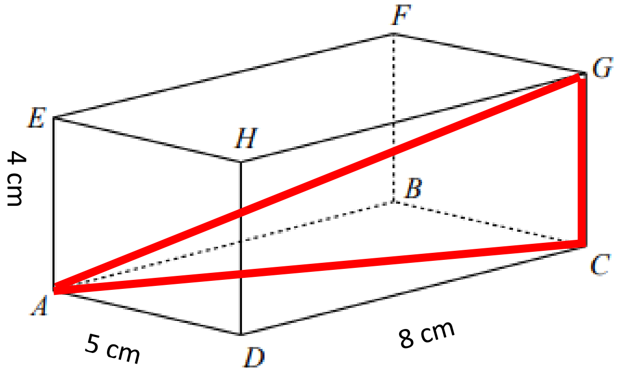
$AE = 4$ cm
$AD = 5$ cm
$DC = 8$ cm

Angle GAC is 40°

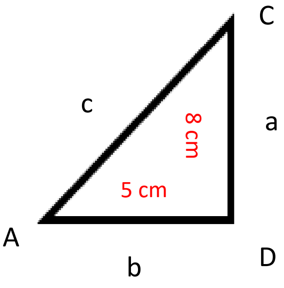


Calculate the length of *AG*.

1st step: Identify the 2D triangle within the 3D shape which includes the side that you need to find the length of



2nd step: Find the length of the base of the triangle using Pythagoras' Theorem.



$c^2 = a^2 + b^2$

$c^2 = 8^2 + 5^2$

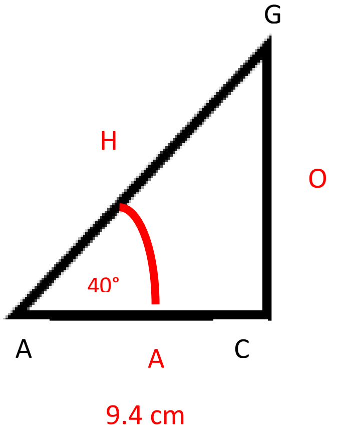
$c^2 = 64 + 25$

$c^2 = 89$

$c = \sqrt{89}$

$c = 9.4$ cm

3rd step: Label the sides of the triangle using opposite, hypotenuse and adjacent and determine which trigonometry function to use from SOHCAHTOA.



4th step: Use trigonometry to find the length of AG.

$\text{Cos }(40) = \frac{Adjacent}{Hypotenuse}$

$\text{Cos }(40) = \frac{9.4}{AG}$

$AG = \frac{9.4}{Cos\ (40)}$

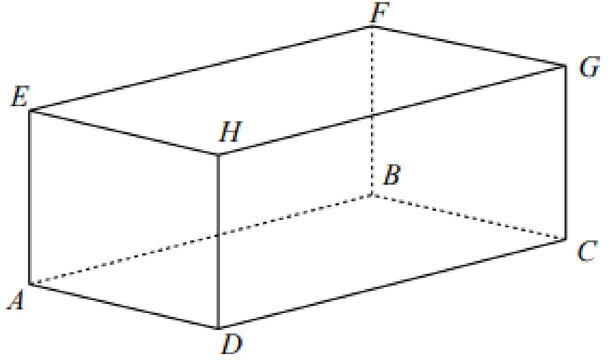
AG = 12.27 cm

3D Trigonometry – calculating the size of an angle

Worked Example.

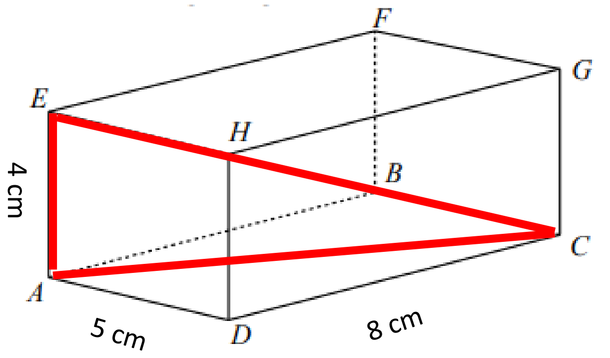
The diagram shows a cuboid *ABCDEFGH*.

AE = 4 cm
AD = 5 cm
DC = 8 cm

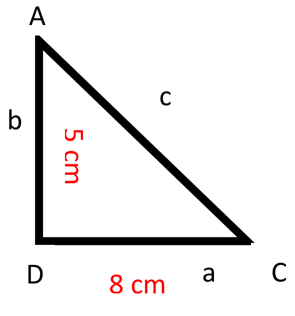


Calculate the size of angle *ECA*.

1st step: Identify the 2D triangle within the 3D shape which includes the angle that you need to find the size of.



2nd step: Find the length of the base of the triangle using Pythagoras' Theorem.



$c^2 = a^2 + b^2$

$c^2 = 8^2 + 5^2$

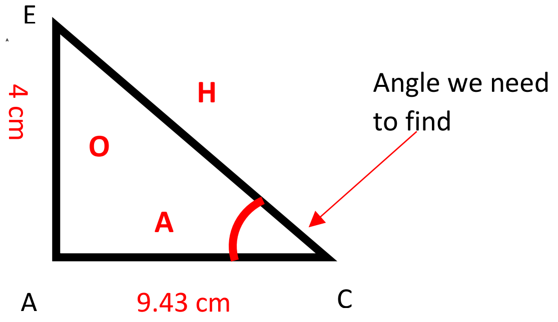
$c^2 = 64 + 25$

$c^2 = 89$

$c = \sqrt{89}$

$c = 9.43$ cm

3rd step: Label the sides of the triangle using opposite, hypotenuse and adjacent and determine which trigonometry function to use from SOHCAHTOA.



Angle we need to find

4th step: Use trigonometry to find the size of the angle.

$\text{Tan } x = \frac{4}{9.43}$

$x = \text{Tan}^{-1}\left(\frac{4}{9.43}\right)$

$x = 22.9855°$

$\underline{x = 23.0°}$

Online Clips

U385, U828, U541, U283, U545, U627, U319, U967, U170

Area of a triangle using trigonometry

Component Knowledge

- Use the area of a triangle formula to find the area of a non-right angled triangle.
- Given the area of a triangle and two sides be able to find a missing angle.

Key Vocabulary

Adjacent	The side next to the angle when using trigonometry
Opposite	The side opposite the angle when using trigonometry.
Sine	A trigonometric function that is equal to the ratio of the side opposite a given angle.
Theta (Θ)	Used to denote a missing angle.

Area of a triangle

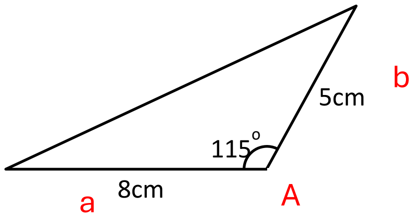
This formula is used when you know 2 sides and the angle between them.

Formula: Area = $\frac{1}{2}absinC$

Example -finding the area

1. Label the sides
2. Substitute into the formula

Find the area of the following triangle.



Area of △ $= ½ \times 8 \times 5 \times \sin(115^{\circ})$

$= \boldsymbol{18.1cm^2}$

Online clips

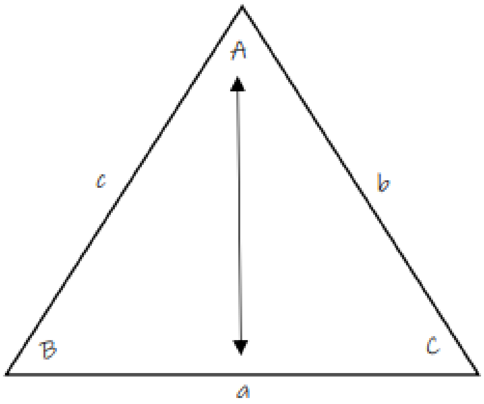
U592

Labelling a non-right angled triangle

Capital letters are used for the 3 angles

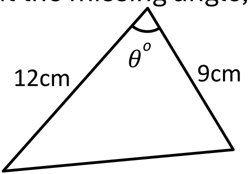
Lower case letters for the 3 sides

Letters of the same type (a and A) are opposite to



Example – Finding the angle given the area

Given that the area of the following triangle is $51.6cm^2$, work out the missing angle, θ°, to one decimal place.



Area of △ **51.6** $= ½ \times 9 \times 12 \times \sin(\theta^{\circ})$

51.6 $= 54\sin(\theta^{\circ})$

$\div 54$ $\div 54$

$\frac{\mathbf{51.6}}{\mathbf{54}} = \sin(\theta^{\circ})$

$sin^{-1}\left(\frac{\mathbf{51.6}}{\mathbf{54}}\right) = \theta^{\circ} = \boldsymbol{72.9^{o}}$

Sine Rule

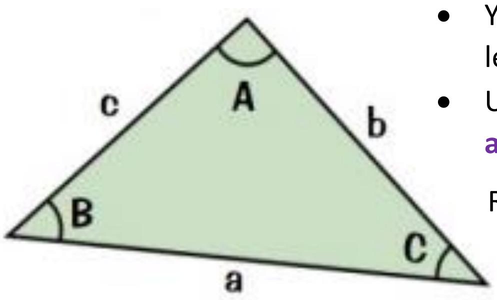
Component Knowledge

- Find a missing side using the sine rule.
- Finad a missing angle using the sine rule.

Key Vocabulary

Sine Rule	Use with non right angled triangles. Use when the question involves two angles and two sides.
Sin/Sine	The ratio of the length of the opposite side to the length of the hypotenuse.
Trigonometry	Trigonometry is the study of triangles: their angles, lengths and more.

Labelling the triangle



- You must label the sides and angles properly so that the letters for the sides and angles correspond with each other.
- Use **lower case letters** for the **sides** and **capitals** for the **angles.**

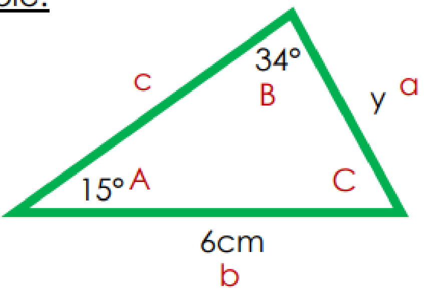
Remember: **Side 'a' is opposite the angle A** etc

It does not matter which sides you decide to call a, b and c, just as long as the angles are then labelled properly.

Sine Rule – Missing Side

Example:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Always label your triangle first

$$\frac{y}{Sin(15°)} = \frac{6}{Sin(34°)}$$

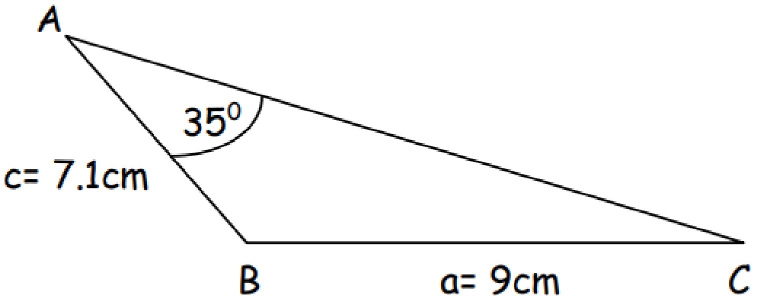
$$y = \frac{6}{Sin(34°)} \times Sin(15°)$$

$$y = 2.7770626 = 2.8cm\ (1d.p.)$$

Sine Rule – Missing angle

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example: To find angle C



$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{7.1} = \frac{\sin 35^0}{9}$$

$$\sin C = \frac{\sin 35^0}{9} \times 7.1$$

$\sin C = 0.4524.....$

$C = \sin^{-1}(0.4524.....)$

$\boldsymbol{C = 28.9^0}$ **(1dp)**

Online clips

U952

Cosine Rule

Component Knowledge

- Identify when to use the cosine rule
- Find the missing side opposite the angle using the cosine rule
- Find a missing angle using the cosine rule

Key Vocabulary

Trigonometry	This the method of finding missing sides and angles in triangles.
θ	Greek letter theta, often used to label missing angles

When do you use the cosine rule

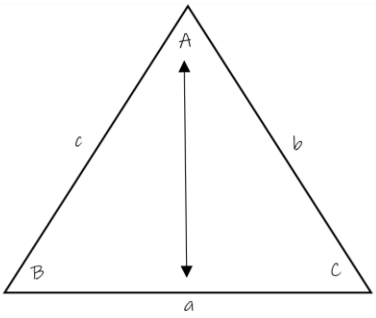
1. For a non-right angled triangle
2. When the problem involves 3 sides and 1 angle

Online clip

U591

Labelling a triangle

- Capital letters are used for the 3 angles
- Lower case letters are used for the 3 sides
- Letters of the same type are opposite each other



Finding a missing side

Formula:

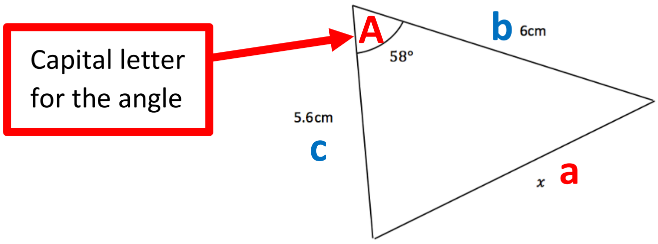
$$a^2 = b^2 + c^2 - 2bc\cos\ A$$

Note

$$2bc\ cosA = 2 \times b \times c \times cosA$$

Example

Find the missing side x



$$a^2 = b^2 + c^2 - 2bc\cos\ A$$

$$x^2 = 6^2 + 5.6^2 - (2 \times 6 \times 5.6 \times \cos 58)$$

$$x^2 = 31.74942544$$

$$x = 5.634662851$$

$$\underline{x = 5.63cm\ (2dp)}$$

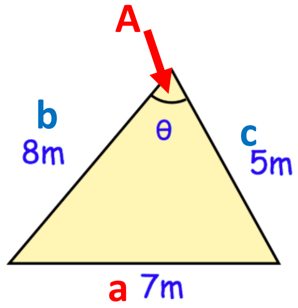
Finding a missing angle

Formula:

$$\cos A = \frac{(b^2 + c^2) - a^2}{2bc}$$

Example

Find the missing angle θ



$$\cos\theta = \frac{(8^2 + 5^2) - 7^2}{2 \times 8 \times 5}$$

$$\cos A = \frac{1}{2}$$

$$A = cos^{-1}(\frac{1}{2})$$

$$\underline{A = 60°}$$

Angles in Polygons

Component Knowledge

- Recognise and name different polygons
- Understand the difference between regular and irregular polygons
- Calculate and use the sum of interior angles
- Know that the sum of any exterior angles of any polygon is 360°
- Know that the interior + exterior angle is 180°

Key Vocabulary

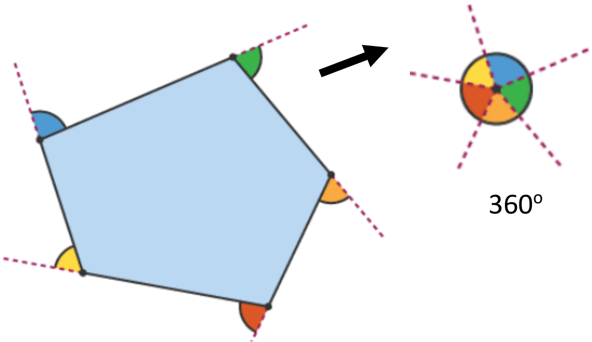
Interior angles	The angles inside the shape
Exterior angles	The angles between the side of a shape and a line extended from the adjacent side
Sum	Total – to add all the angles together
Polygon	A 2D closed shape made with straight lines
Regular	When all the sides are the same length and all angles are the same
Irregular	Shape with sides of different lengths and angles of different sizes

Exterior angles

The **sum of exterior angles** in any polygon is **360°**

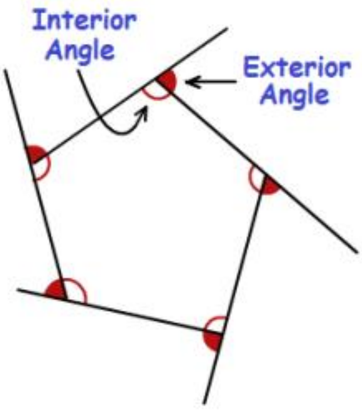
The size of each exterior angle in a regular polygon is **360° ÷ number of sides**

This can be rearranged to **number of sides = 360 ÷ angle**

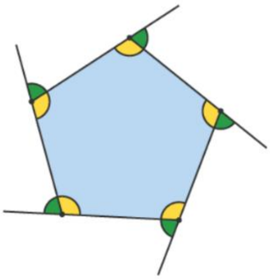


Interior and Exterior angles

Interior + exterior angle = 180°



Example 1 – Calculate the interior and exterior angle of a regular pentagon.



Exterior angle

= 360 ÷ 5 = 72°

Interior angle = 180 -72 = 108°

Example 2 – A regular polygon has exterior angles of 20°. How many sides does it have?

Exterior angle = 360 ÷ number of sides

Number of sides = 360 ÷ 20 = 18

= 18 sides

Interior angles in regular polygons

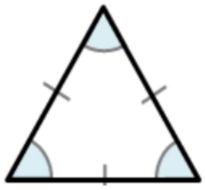
Sum of interior angles = (n – 2) x 180

Where n is the number of sides.

Each interior angle on a regular shape =

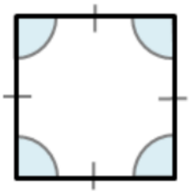
Total interior angles ÷ number of sides

Triangle



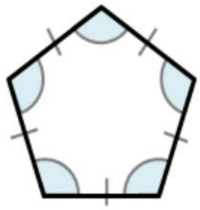
Number of sides	3
Sum of interior angles	180°
Size of each interior angle	60°

Square



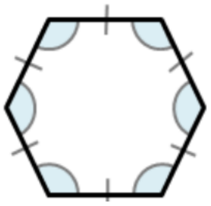
Number of sides	4
Sum of interior angles	360°
Size of each interior angle	90°

Pentagon



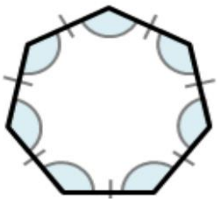
Number of sides	5
Sum of interior angles	540°
Size of each interior angle	108°

Hexagon



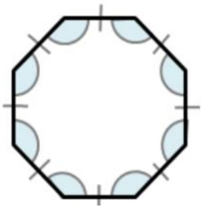
Number of sides	6
Sum of interior angles	720°
Size of each interior angle	120°

Heptagon



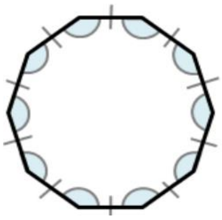
Number of sides	7
Sum of interior angles	900°
Size of each interior angle	128.6° (1dp)

Octagon



Number of sides	8
Sum of interior angles	1080°
Size of each interior angle	135°

Decagon



Number of sides	10
Sum of interior angles	1440°
Size of each interior angle	144°

n Sided Shape

Number of sides	n
Number of interior angles	(n- 2)x 180°
Size of each interior angle	$\frac{(n-2) \times 180°}{n}$

Irregular polygons

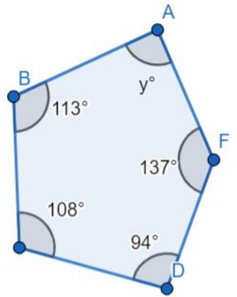
Example –Find the value of y

5 sides, irregular polygon

Sum of interior angles = (5 -2) x180 = 540°

113 + 108 + 137 + 94 = 452

540 – 452 = 88 **x = 88°**



Online clips

U628, U732, U329, U427