Angles on Parallel lines

Component Knowledge

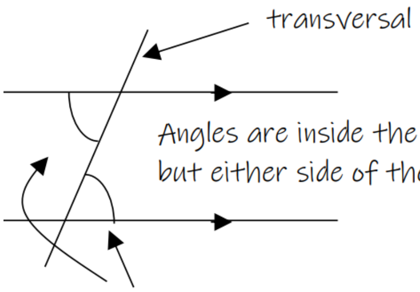
- Basic angle facts – such as angles on a straight line = $180°$
- Recognise that a traversal is a line which crosses a set of parallel lines
- To be able to find missing angles on parallel lines.

Key Vocabulary

Parallel lines	Lines are parallel if they are always the same distance apart (called "equidistant"), and will never meet
Supplementary	Two angles are supplementary when they add up to 180˚

Parallel Lines Angle Facts:

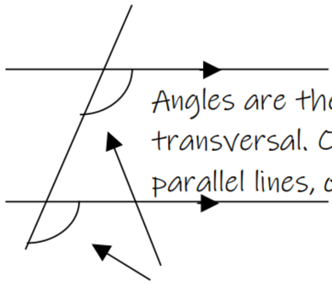
Alternate Angles



Angles are inside the parallel lines but either side of the transversal

Alternate angles are equal (Z shape)

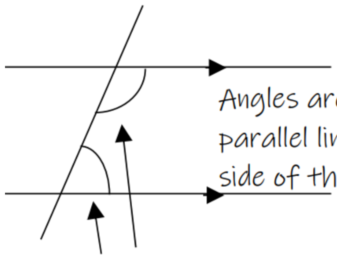
Corresponding angles



Angles are the same side of the transversal. One inside the parallel lines, one outside.

Corresponding angles are equal (F shape)

Co-Interior Angles

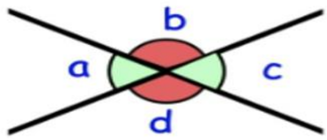


Angles are both inside the parallel lines and on the same side of the transversal.

Allied angles sum to 180° (C shape)

These are also called co-interior angles

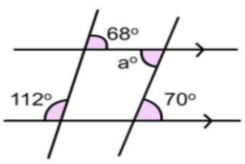
Vertically opposite angles



Vertically opposite angles are equal

a = c and b = d

Examples



a = 70˚

Reason -Alternate Angles are equal

Important! ALWAYS state the angle and the reason.

Online Clips

U390, U730, U628, U732, U655, U826

Bearings

Component Knowledge

- To be able to understand the 3 rules of bearings and use this to measure and draw bearings.
- To be able to use angle facts to find missing bearings.

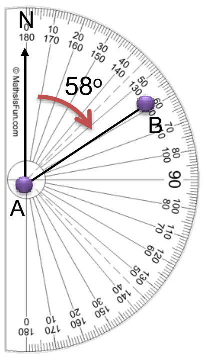
Key Vocabulary

Bearing	A measure of direction, it is used to represent the direction of one point relative to another. It is the angle in degrees measured clockwise from north. Always written in three-figures.
Protractor	The instrument used for measuring angles (measured in degrees).
Scale	Used to reduce real world dimensions to a useable size.

Measuring and drawing Bearings using a protractor:

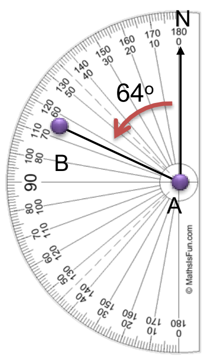
Bearings are measured and drawn

- From the **North** (N)
- **clockwise**
- are always written as **3 figures.**



Bearing = 058°

To measure a bearing greater than 180°, measure the angle anticlockwise and subtract from 360°.



Bearing = 360° – 64° = 296°

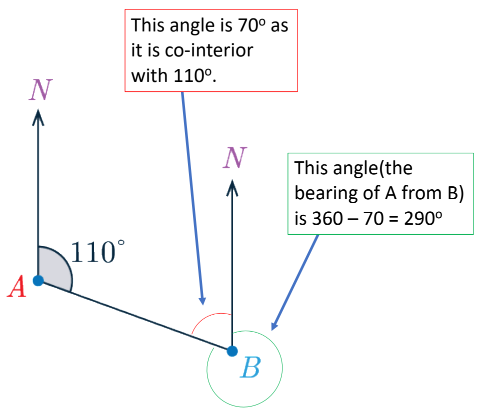
Be careful where a bearing is being measured from. If you were measuring the bearing of B from A your protractor would be on A.

Example: Bearings without a protractor

We are given the bearing of B from A. To calculate the area of A from B we can use angle facts.

Co-interior angles add up to 180°.

Angles at a point equal 360°.



Online clips

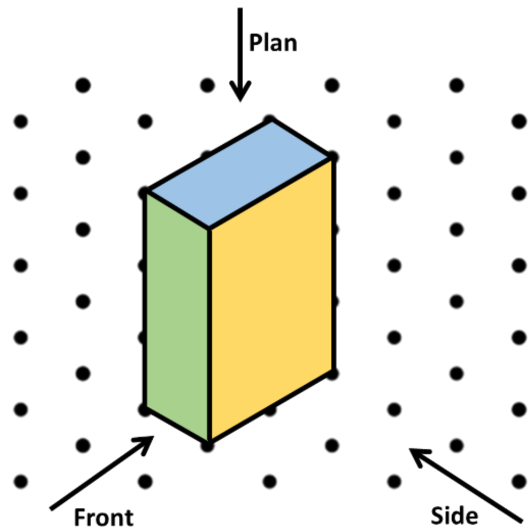
Plans and elevations

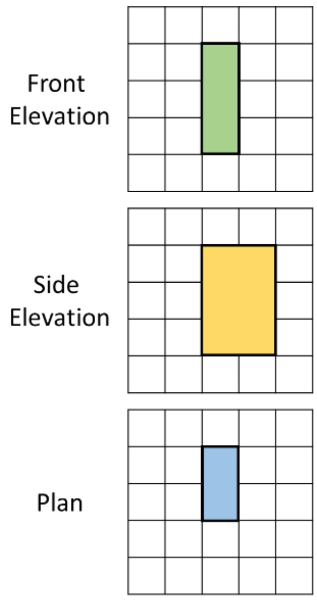
Component Knowledge

- Draw the plan of an oriented 3-dimensional shape
- Draw the front elevation (direction specified) of a 3-dimensional shape
- Draw the side elevation (direction specified) of a 3-dimensional shape

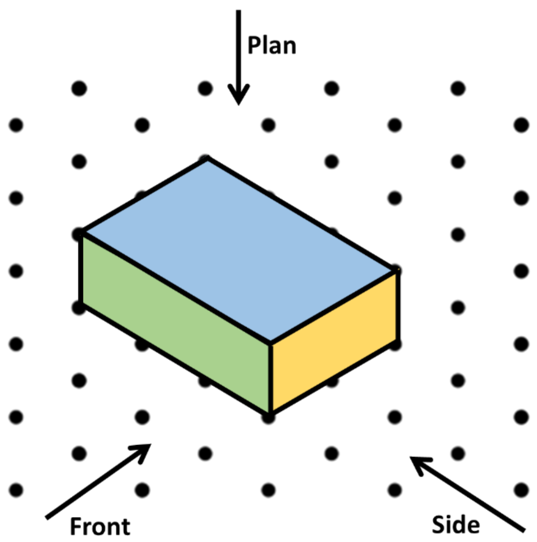
Key Vocabulary

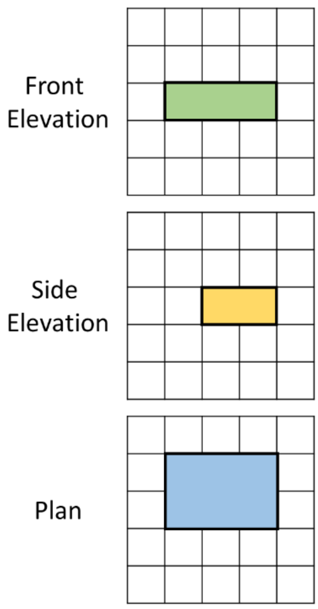
Plan	The view of an oriented 3-dimensional shape from above
Front elevation	The view of an oriented 3-dimensional shape from a specified front direction
Side elevation	The view of an oriented 3-dimensional shape from the side





Ensure that the dimensions of the plan and the elevation are consistent with the lengths of the 3-dimensional shape.



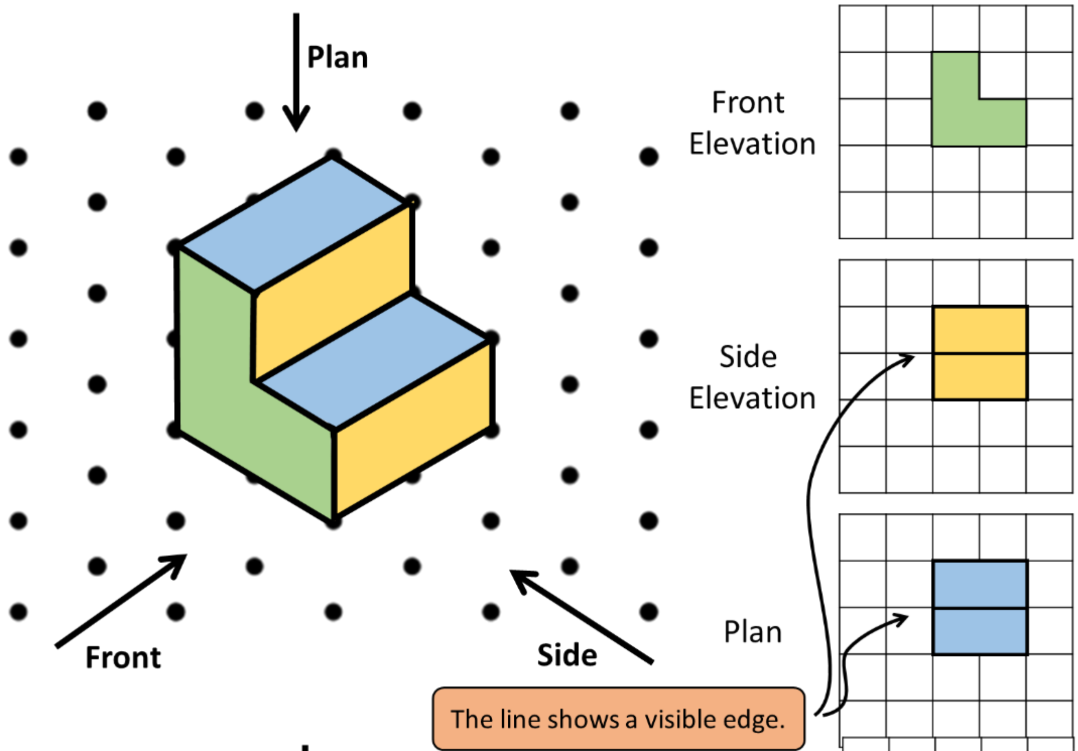


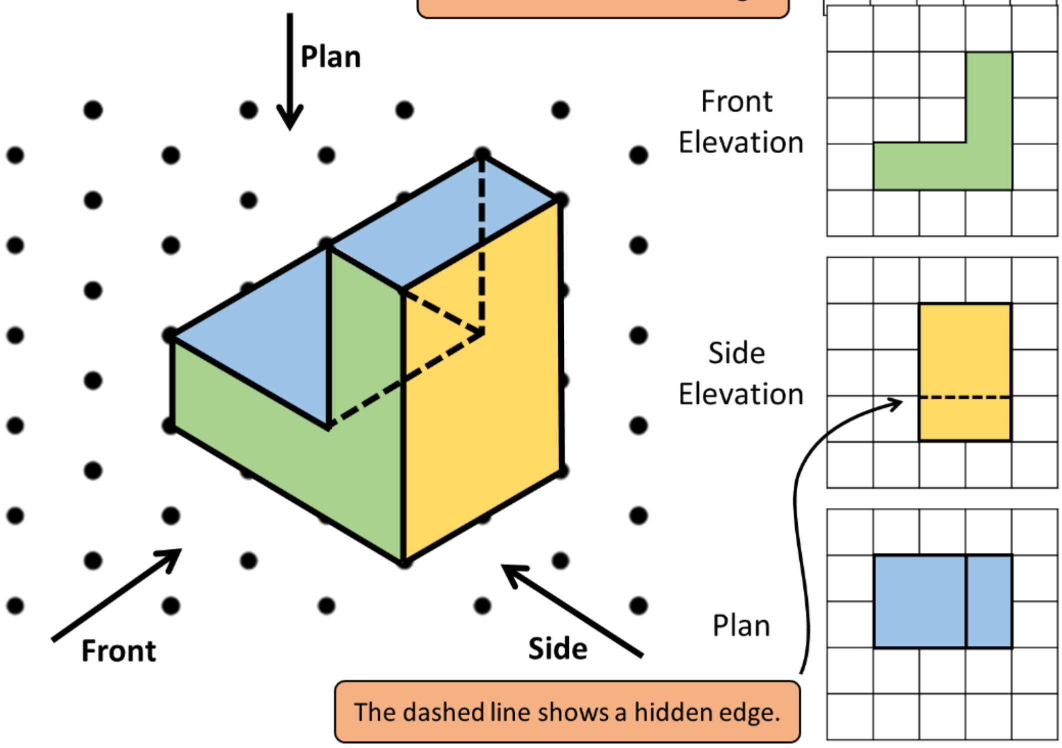
If only the front direction is specified, both the left and right-side view are acceptable as the side elevation.

(They are either the same, or mirror images)

Showing edges in plans and elevations

This provides more information about the shapes and makes it easier to identify the direction from which the plan and elevation are drawn.





Online clips

M229

Isometric Drawing

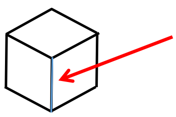
Component Knowledge

- To be able to draw a 3D shape on Isometric paper

Key Vocabulary

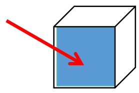
Isometric	An isometric drawing is a drawing of a 3- dimensional shape on a two-dimensional surface. A vertical line is used as a place to start. Horizontal lines are created at 30- degree angles.
Isometric Paper	Isometric paper is paper with dots arranged in equilateral triangles.
Edge	An edge is where two faces, on a shape, come together. On 3D shapes they are the lines that separate each face.
Vertex	A vertex is a corner where edges meet.
Faces	A face is a flat or curved surface on a 3D shape.

We can draw 2D representations of 3D shapes from two different angles:

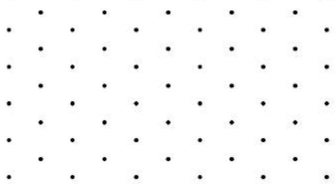


This one has a front **edge**

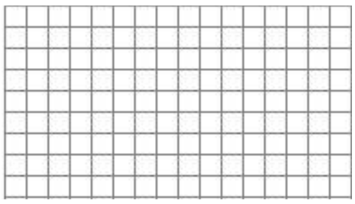
This one has a front **face**



We can draw cubes from this angle on isometric paper (spotty triangle paper)



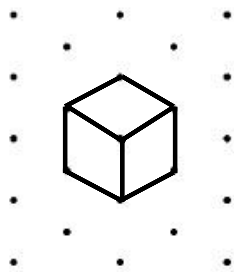
We can draw cubes from this angle on square paper.



To draw a single cube on the isometric paper.

Make sure you have the paper this way with the dots going down, not across

The lines can never be drawn horizontally.

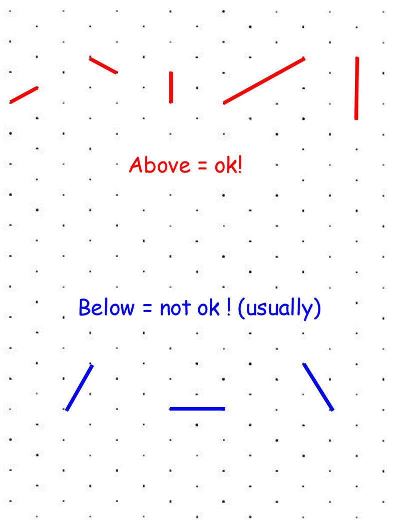


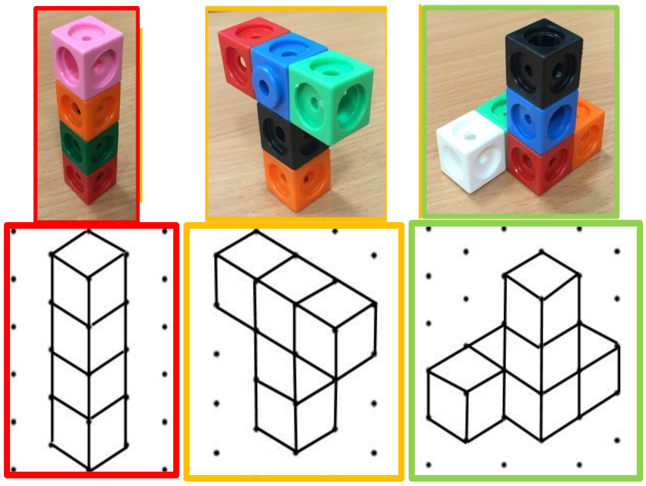
Start by drawing just one face

Then complete the cube

When drawing objects on isometric paper, you very rarely (if ever) join dots across wider gaps

They usually join to dots directly next to them…





Online clips

Straight line graphs

Component Knowledge

- Recognise and sketch horizontal and vertical graphs
- Complete a table of values
- Plot straight line graphs
- Identify gradients/intercepts from a graph
- Identify gradients/intercepts from an equation

Key Vocabulary

Axis	A fixed reference line a grid to help show the position of coordinates
Gradient	How steep a graph is at any point
Y intercept	Where the graph cuts through the y axis
Coordinate	A set of values that show an exact position
Quadrant	Any of the 4 areas made when we divide up a plane by an x and y axis
Vertical	In an up and down position. The y axis is the vertical axis
Horizontal	Going side to side. The x axis is the horizontal axis
Graph	A diagram showing the relationship between two quantities

Completing a table of values and plotting a graph

To plot a straight line graph, you may be given a table or you may need to draw one.

Example: Plot the graph of $y = 4x - 2$ for the values of x from -3 to 3.

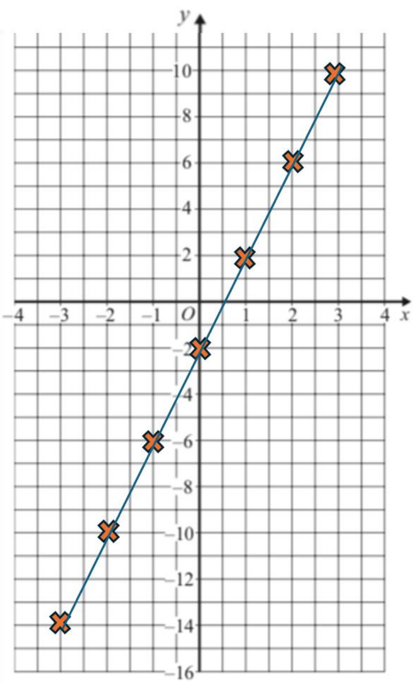
1) Draw a table of values if you have not been given one.

x	-3	-2	-1	0	1	2	3
y							

2) Substitute in your x values to $y = 4x - 2$, this will give the corresponding y values.

x	-3	-2	-1	0	1	2	3
y	-14	-10	-6	-2	2	6	10

3) Plot the points on the graph.
E.g. (-3, -14), (-2, -10), (-1, -6), (0, -2), ... etc



4) Join up with a straight line.

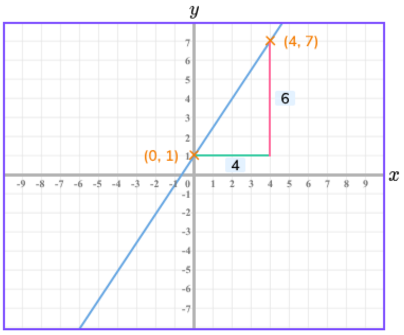
The equations of all straight lines can be written in the form:

$$y = mx + c$$

Gradient – The number in front of the x. This tells us how steep the line is.

Intercept – The number on its own. Shows where the line cuts the y axis.

The gradient of a line tells us how steep the line is, the greater the gradient the steeper the line.



You can find the gradient using the graph by picking 2 points on the line and using

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

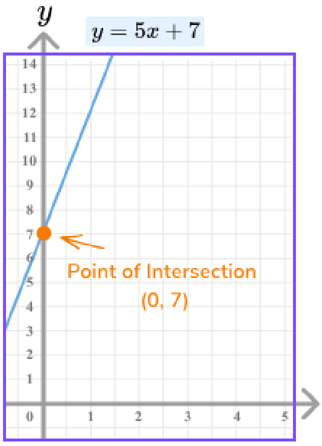
The change in y is equal to $y_2 - y_1 = 7 - 1 = 6$

The change in x is equal to $x_2 - x_1 = 4 - 0 = 4$

$$m = \frac{6}{4}$$

The y intercept is where the line crosses the y axis

You can find the y intercept from the equation by putting x equal to 0



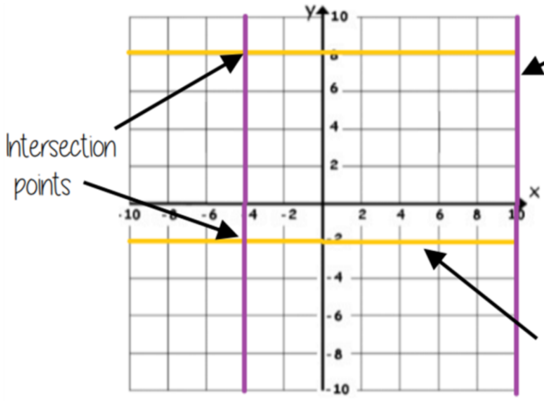
The gradient and intercept of a straight line can also be identified form the formula.

Example: Find the gradient and intercept of the following lines.

1)	$y = 5x - 2$		Grad = 5	Intercept = - 2
2)	$2y = 4x + 5$	$y = 2x + 2.5$	Grad = 2	Intercept = 2.5
3)	$x + y = 10$	$y = -x + 10$	Grad = - 1	Intercept = 10

Rearrange all equations so they are in the form y = mx + c (the y must be isolated)

Lines parallel to the axis (Horizontal and Vetical lines)



All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

e.g. (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

Online clips

M797, M932, M544, M888

Approximate Solutions to Equations Using Graphs

Component Knowledge

- Be able to draw straight line graphs.
- Be able to approximate solutions from graphs.

Key Vocabulary

Graph	A diagram showing the relationship between two quantities
Equation	A mathematical statement showing that two expressions are equal
Approximate	Close to an actual value but may not be completely accurate
Solution	Values for which an equation is true
Linear	Increasing or decreasing at a constant rate to form a straight line
Co-Ordinates	A pair of numbers used to describe the position of a point
Axis/Axes	A fixed reference line on a grid to help show the position of co-ordinates
Plot	To represent the relationship between two quantities graphically
Simultaneous	When two or more things occur at the same time
Intersect	When two or more things pass through each other

Equations describe a relationship between two values. We can plot this relationship on a graph to help us visualise the relationship more easily. By doing this we can also approximate solutions for given equations.

The equations of all straight lines can be written in the form:

$$y = mx + c$$

Gradient – The number in front of the x. This tells us how steep the line is.

Intercept – The number on its own. Shows where the line cuts the y axis.

Plotting Linear Graphs

Draw the graph of: $y = 2x + 1$

x	-1	0	1	2	3
y	-1	1	3	5	7

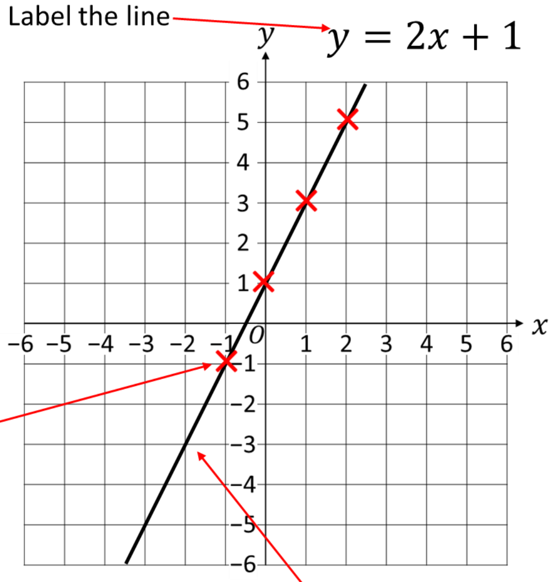
$(-1,-1)$ $(0,1)$ $(1,3)$ $(2,5)$ $(3,7)$

$E.g. (2 \times 2) + 1 = 5$

1) Complete the table of values by substituting x values.

2) Plot each pair of values as coordinates.

4) Label the line

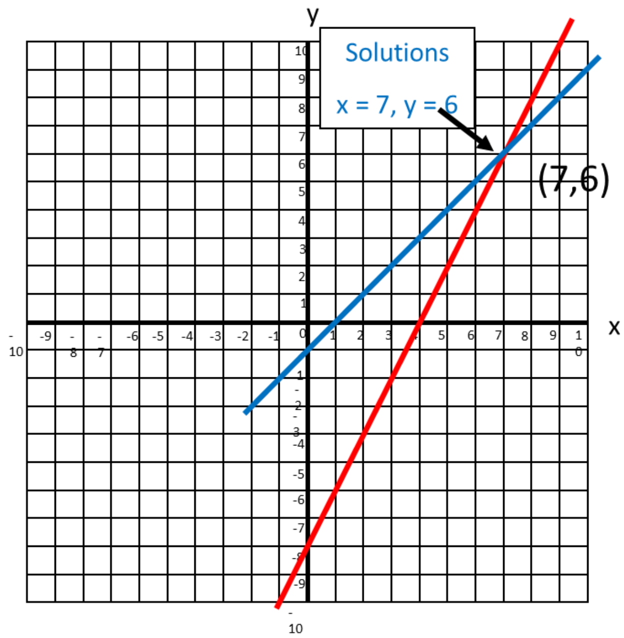


3) Join the points to make a line.

Solving Simultaneous Equations When the Graph is Given

Solve the simultaneous equations

$2x - y = 8$ and $x - y = 1$



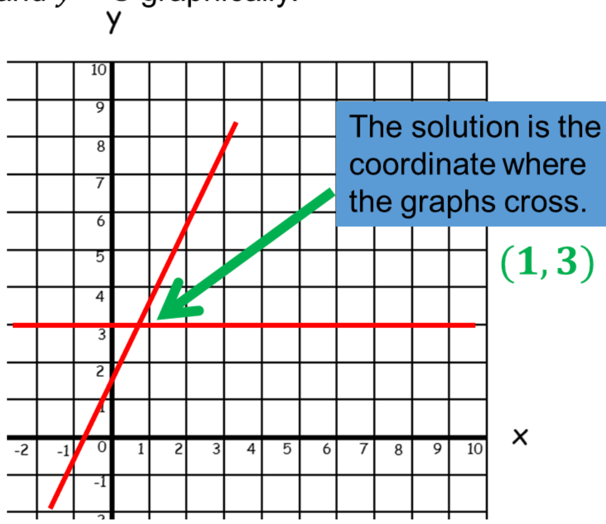
The solution to the simultaneous equation is where the two lines intersect.

There will always be two solutions one for x and one for y.

So $x = 7$ and $y = 6$.

Plotting and Solving Graphically

Solve the simultaneous equations $y = 2x + 1$ and $y = 3$ graphically:



Sometimes we are asked to show the solution graphically but not given the graphs.

In these cases we must first plot the graphs as shown previously.

First sketch $y = 3$.

Draw a table of values to sketch $y = 2x + 1$ and plot the line

x	0	1	1
y	1	3	5

So $x = 1$ and $y = 3$.

Online clips

U836

Simultaneous linear equations

Component Knowledge

- Solving simultaneous linear equations with a balanced variable by elimination
- Solving simultaneous linear equations where balancing a variable is required
- Form and solve simultaneous equations.

Key Vocabulary

Simultaneous equations	Two or more equations that are two be solved (if possible) by using the *same* value for each variable
Coefficient	The number factor in an algebraic term, multiplied with variables (e.g. 4 in $4x$)
Balancing variables	Equating the coefficients of like terms in different equations by multiplying with suitable factors
Eliminating variables	Reducing the term containing a particular variable in an equation to 0 by subtracting/adding another equation with the same/opposite term
Substitution	Assigning a value to a variable (e.g. substituting $y = 8$ in $6y$ gives 48)

Solving simultaneous equations – no balancing needed

In the first example, because the two equations have **equal** terms in x – both are $3x$ – *subtracting* the equations (remember to subtract both sides) *eliminates* the x term. The resulting equation has only one unknown, y, and can be solved.

$$3x + 4y = 29$$
$$-\quad 3x + 2y = 19$$
$$2y = 10$$
$$y = 5$$

Substitute y into either equation to find x.

$$3x + (2 \times 5) = 19$$
$$3x + 10 = 19$$
$$3x = 9$$
$$x = 3$$

Here the value found for y is **substituted** into the second equation to obtain an equation in terms of x. The first equation could have been used too.

Whichever equation is used for substitution, it is good practice to check the pair of values found in the other equation too, to ensure no mistakes have been made:

$$3 \times 3 + 2 \times 5 = 19$$

In the second example, because the two equations have **opposite** terms in y – one is $2y$ and the other $-2y$ – *adding* the equations eliminates the y term.

$$3x + 2y = 16$$
$$+\quad 2x - 2y = 4$$
$$5x = 20$$
$$x = 4$$

Substitute x back in to find y.

$$(2 \times 4) - 2y = 4$$
$$8 - 2y = 4$$
$$8 = 4 + 2y$$
$$4 = 2y$$
$$2 = y$$

Forming simultaneous equations – balancing a variable

$$2x + 8y = 32$$
$$x + 3y = 13$$

×2 ×2

$$2x + 6y = 26$$
$$2y = 6$$
$$y = 3$$

$$2x + 6(3) = 26$$
$$2x + 18 = 26$$
$$2x = 8$$
$$x = 4$$

Here neither the x nor the y terms are already balanced. But the x terms can be balanced by multiplying the second equation by 2. (Remember to **multiply both sides** by the factor.)

The modified second equation can then be subtracted from the first, and the subsequent steps are as before.

$$5x + 4y = 19$$
$$2x - 3y = 3$$

×3 ×4 ×4 ×3

$$8x - 12y = 12$$
$$15x + 12y = 57$$
$$23x = 69$$

$$x = 3$$

$$15 + 4y = 19$$
$$4y = 4$$
$$y = 1$$

In this example the y terms can be balanced by multiplying the first equation by 3 and the second by 4, since 12 is the lowest common multiple of the starting coefficients. (Alternatively, we can balance the x terms. What factors would be needed in that case?)

The modified equations are then added – since the y terms have opposite signs – and the following steps are as before.

Forming simultaneous equations to solve a problem

Barry buys 200 pieces of stationery for £76.

Of the 200 pieces of stationery, x of them are rulers that cost 50p each and y of them are pens that cost 20p each.

Find how many rulers and pens Barry buys.

The information in the question can be written as the simultaneous equations

$x + y = 200$

$50x + 20y = 7600$ (amounts are written in pence)

Multiply the first equation by 50 to give $50x + 50y = 10000$. The x terms are now balanced, and subtracting the second equation gives $30y = 2400$.

Therefore $y = 80$, and using the first equation $x = 120$.

Online clips

U760

Congruency

Component Knowledge

- Know what congruency is and how to identify whether shapes are congruent
- Recognise congruent triangles
- Know the rules for congruency

Key Vocabulary

Congruent	The same shape and size
Triangle	A 3 sided flat shape with straight sides
Hypotenuse	The side opposite the right angle in a right angles triangle
Right angle	An angle which is equal to 90°
Identical	Exactly the same
Side	One of the line segments that make a flat 2D shape

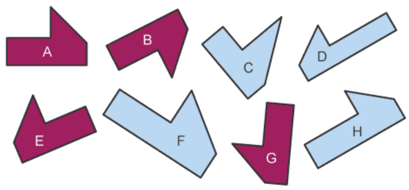
Key Concepts

Shapes are **congruent** if they are **identical** – same shape and same size.

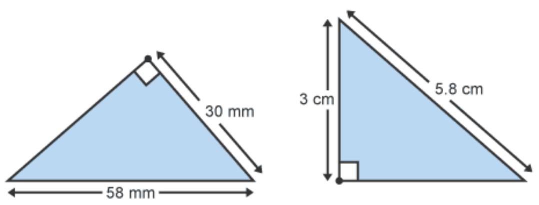
Shapes can be rotated or reflected but still be congruent.

Examples

Which shapes are congruent?



Shapes A, B, E and G are congruent as they are identical in size and shape.



These are congruent, they both have a right angle, the same hypotenuse and another side the same

Triangles

There are four ways of proving that two triangles are congruent:

1) **SSS** (Side, Side, Side)
    a. All 3 sides are the same in both triangles
2) **RHS** (Right angle, Hypotenuse, Side)
    a. Both triangles have a right angle, the same hypotenuse and one other side the same
3) **SAS** (Side, Angle, Side)
    a. Two sides with the angle in between them are the same in both triangles
4) **ASA** (Angle, side, Angle) or **AAS**
    a. One side and two angles are the same in both triangles

Misconceptions

Proving all 3 angles are the same is **not** proving they are congruent, as one could be an enlargement of the other.

Angle, Side, Side is **not** a proof for congruency as the angle needs to be contained between the two sides.

Online clips

U790, U112, U866

Similar shapes

Component Knowledge

- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles in similar shapes
- Understand similarity & congruence

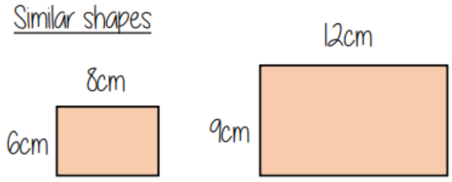
Key Vocabulary

Enlarge	Make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale factor	The multiplier of enlargement
Similar	When one shape can become another through a reflection, rotation, enlargement of translation
Corresponding	Items that appear in the same place in two similar situations

Identifying similar shapes

Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180^0

Similar shapes



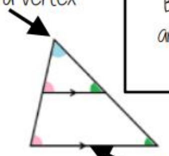
Scale Factor:
Both sides on the bigger shape are 1.5 times bigger

Compare sides: 6 : 9 → 2 : 3 8 : 12 → 2 : 3

Both sets of sides are in the same ratio

Similar triangles

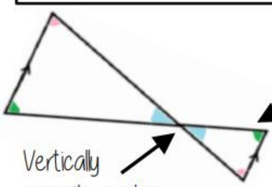
Shares a vertex



Because corresponding angles are equal the highlighted angles are the same size

Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

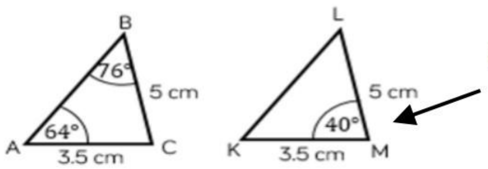


All the angles in both triangles are the same and so similar

Vertically opposite angles

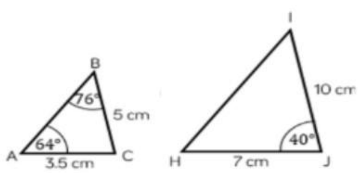
Congruence & similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



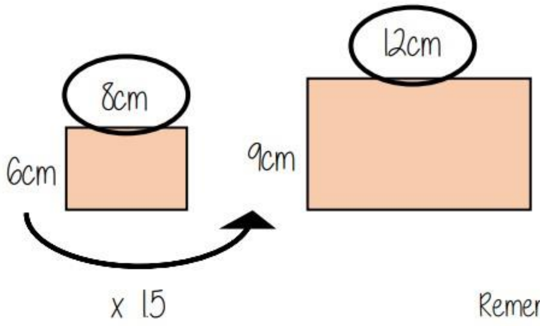
AĈB = KM̂L

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Information in similar shapes



Compare the equivalent side on both shapes

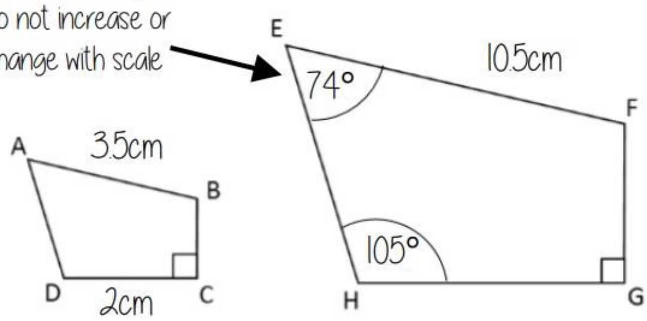
Scale Factor is the multiplicative relationship between the two lengths

Remember angles do not increase or change with scale

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

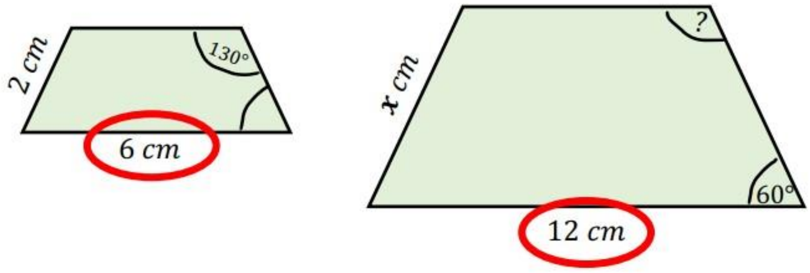
AB and EF are corresponding



Further example

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



Corresponding sides identify the scale factor $\frac{12}{6} = 2$ Scale Factor = **2**

Calculate the missing side Length (corresponding side) x scale factor

$2cm \times 2$

$x = 4cm$

Enlargement does not change angle size

Calculate the missing angle Corresponding angles remain the same

130°

Online clips

M124, M377, M324, M606

Enlargement

Component Knowledge

- Enlarge a rectilinear shape by a given positive scale factor
- Enlarge a rectilinear shape, given a positive integer scale factor and a centre
- Enlarge a rectilinear shape, given a positive fractional scale factor and a centre
- Describe an enlargement in terms of scale factor and centre

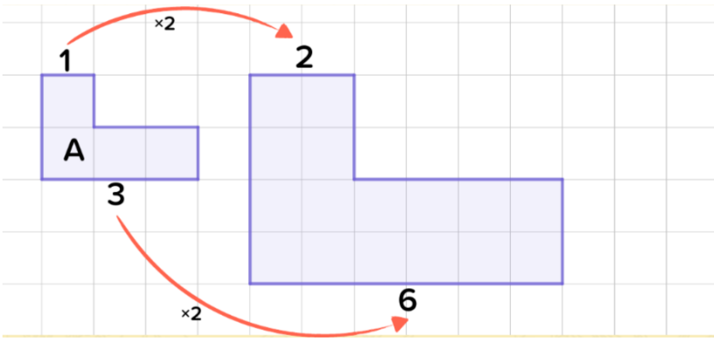
Key Vocabulary

Enlargement	A transformation of a shape in which all dimensions are multiplied by the same number
Scale factor	The number by which dimensions are multiplied in an enlargement
Centre of enlargement	The point from which distances to the *object* and the *image* of an enlargement are measured

Enlarging by a scale factor

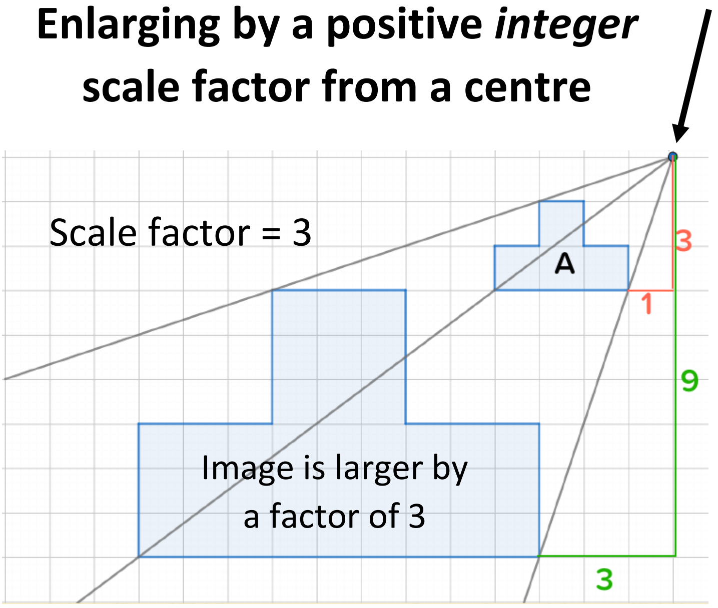
In an *enlargement* all dimensions are multiplied by the same number, called the **scale factor**. In this example shape A has been enlarged by scale factor 2.

If the scale factor is smaller than 1 the dimensions are in fact reduced (divided), although the transformation is still called an enlargement! (See next page)

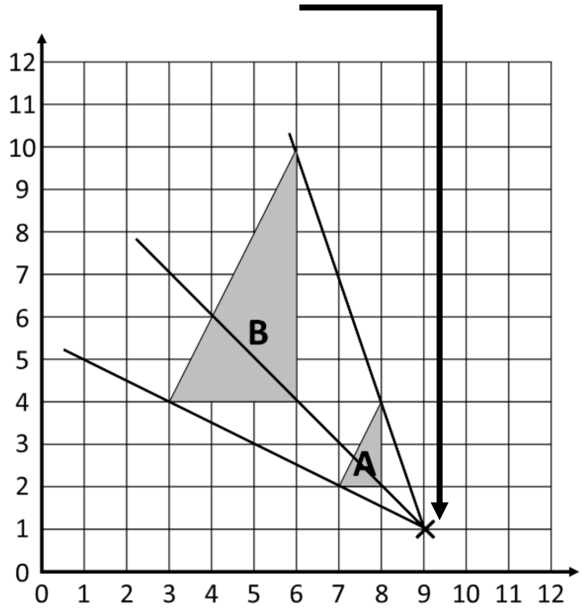


Enlarging by a positive *integer* scale factor from a centre

Centre of enlargement



Measure the distance from the centre of enlargement to each vertex of the *object* shape A; the corresponding vertex in the *image* is triple that distance in the **same** direction

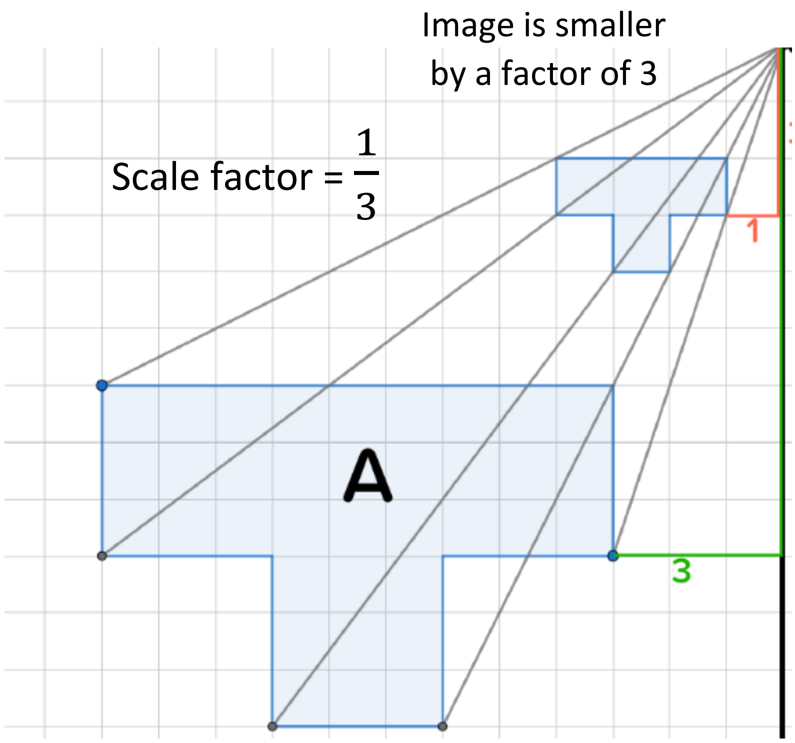


If the object shape is drawn on a coordinate grid, the centre may be specified by coordinates (here the centre is (9,1))

Enlarging by a positive *fractional* scale factor from a centre

A positive scale factor that is smaller than 1 reduces the dimensions of the object shape.

Here the distance from the centre of enlargement to each vertex of the object shape A is measured and then **divided** by 3 to find the corresponding vertex in the image (still in the same direction)



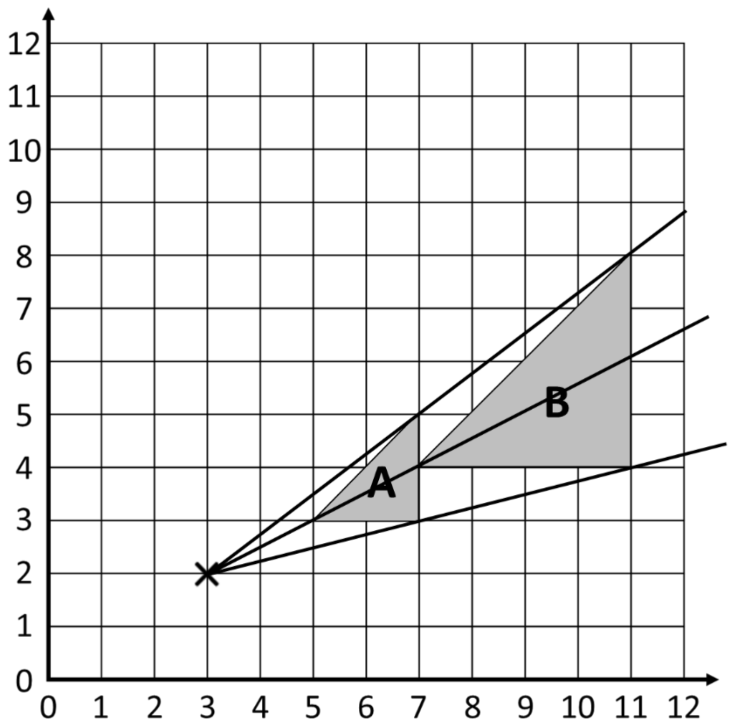
Describing an enlargement

An enlargement is easily identified as such by the change in dimensions.

To determine the scale factor, calculate the ratio of the lengths of corresponding sides in the object and its image.

For the centre, draw lines through two pairs of corresponding vertices and find their point of intersection (thus retracing the steps of the process of enlarging)

The enlargement shown here – from A to B – has scale factor 2 and centre (3,2)



<u>Online clips</u>

M178, U519