Pythagoras Theorem

Component Knowledge

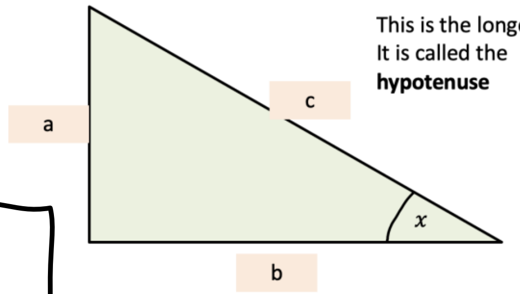
- Identify the hypotenuse in a right-angled triangle.
- Use substitution in formula.
- Solve an equation by rearranging

Key Vocabulary

Hypotenuse	The longest side in a right-angled triangle
Opposite	The side facing the given angle in a right-angled triangle
Adjacent	The side next to the given angle in a right-angled triangle
Square number	The result when you multiply a number by itself.

Key Properties:

This is the **opposite** side to "angle x"



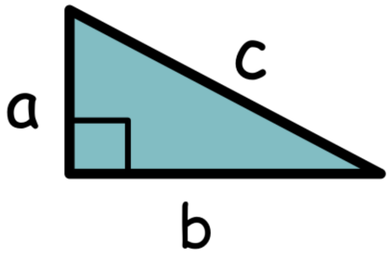
This is the longest side. It is called the **hypotenuse**

This is the **adjacent** side to "angle x"

Note – the hypotenuse is always the side facing the right angle

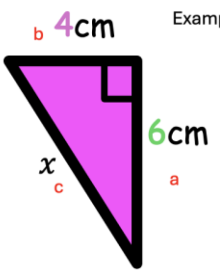
The formula:

$$a^2 + b^2 = c^2$$



Using Pythagoras Theorem to find missing sides

Finding the hypotenuse:



Example: Find x to 1dp

- Step 1 – Label all the sides
- Step 2 – substitute values into the formula

$a^2 + b^2 = c^2$

$6^2 + 4^2 = c^2$

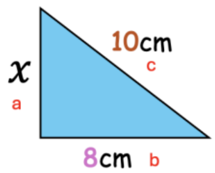
Solve the equation

$36 + 4 = c^2$

40	=	c^2
$(\sqrt{\ })$		$(\sqrt{\ })$
6.426	=	c

- Step 3 – round the number and don't forget your units
- x = 6.4cm (to 1dp)

Finding a shorter side:



Example: Find x to 1dp

- Step 1 – Label all the sides
- Step 2 – substitute values into the formula

$a^2 + b^2 = c^2$

$a^2 + 8^2 = 10^2$

Solve the equation

$a^2 + 64$	=	100
(-64)		(-64)
a^2	=	36
$(\sqrt{\ })$		$(\sqrt{\ })$
a	=	6

- Step 3 – round the number and don't forget your units
- x = 6cm

Online Clips

U385, U828

Trigonometry

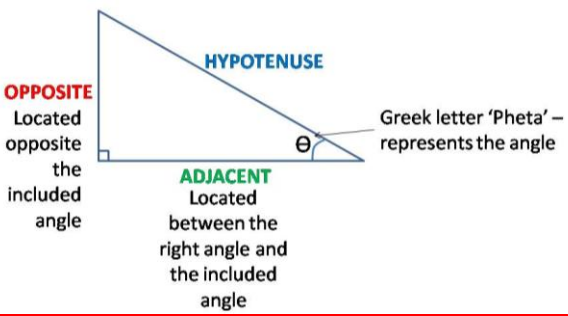
Component Knowledge

- Recall the three trigonometric ratios
- Correctly label the sides of a right-angled triangle with Opp, Adj, Hyp
- Identify the correct trigonometric ratios to use.
- Use the correct trigonometric ratio to find the missing side or angle.
- To identify exact values for key angles using trigonometric ratios.

Key Vocabulary

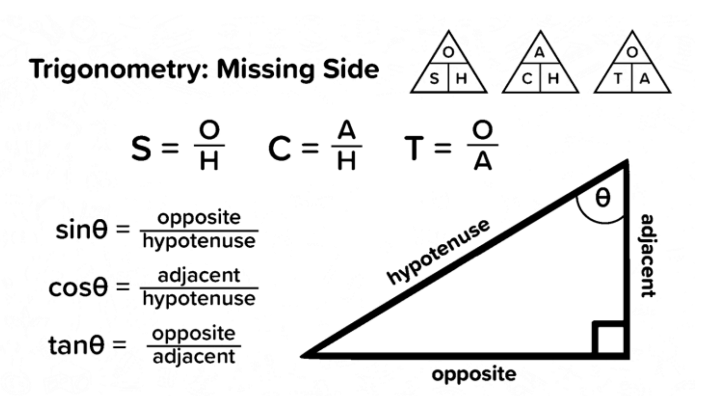
Trigonometry (trig)	Trigonometry helps us find angles and lengths in right-angled triangles.
Trigonometric ratios	There are three trigonometric ratios, depending on the position of the unknown sides and angles.
Hypotenuse side (Hyp)	The length of the longest side of a right-angled triangle.
Opposite side (Opp)	The length of the side opposite the given angle.
Adjacent side (Adj)	The length of the side next to the given angle and right-angle.
Sine ratio (sin)	Is used to describe the relationship between the given angle, Opposite side and Hypotenuse
Cosine ratio (cos)	Is used to describe the relationship between the given angle, Adjacent side and Hypotenuse
Tangent ratio (tan)	Ratio used to describe the relationship between the given angle, Opposite side and Adjacent side.

Labelling the sides



Hypotenuse will **always** be opposite the 90° angle. The Opposite and Adjacent will change depending on where the given angle (Θ) is on the diagram.

Trig ratios



Finding the missing side

- numerator

Remember to show your full calculator answer too, should you have a decimal, and then round to get your final answer.

To find the length of b

Step 1: Label each side

Step 2: Choose the correct formula

Since we know the hypotenuse and need to find the adjacent side, we can use cos

Step 3: Substitute the values into the formula

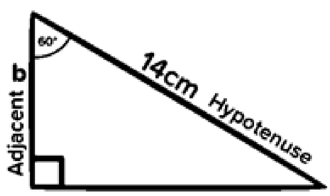
$\cos(60) = \frac{b}{14}$

Step 4: Rearrange to find b

$b = \cos(60) \times 14$

b = 7

SOH CAH TOA



Finding the missing side- denominator

Q Find the length of h

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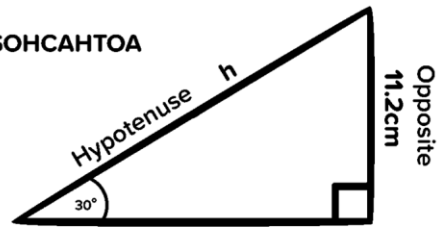
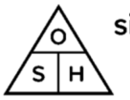


Step 1 Label each side

Step 2 Choose the correct formula

Since we know the opposite side and need to find the hypotenuse, we can use sin



$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

Step 3 Substitute the values into the formula

$\sin(30) = \dfrac{11.2}{h}$

Step 4 Rearrange to find h

$h = \dfrac{11.2}{\sin(30)}$

A 22.4cm

Here we have multiplied both sides by h and then divided both sides by sin (30)

Finding the missing side- angle

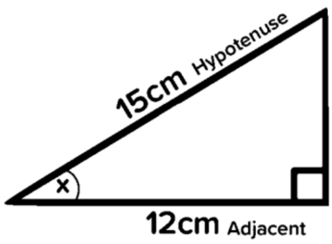
To type $\sin^{-1}$ into your calc, press SHIFT and then the SIN button.

To type $\cos^{-1}$ into your calc, press SHIFT and then the COS button.

To type $\tan^{-1}$ into your calc, press SHIFT and then the TAN button.

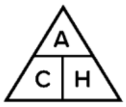
Q Calculate the value of the missing angle x

Give your answer to 1 decimal place.



Step 1 Label the triangle

Step 2 Choose the correct formula



$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

Step 3 Substitute and solve

$\cos(x) = \dfrac{12}{15}$

$x = \cos^{-1}\left(\dfrac{12}{15}\right)$

$= 36.8698...$

1 decimal place

A 36.9°

Exact trigonometric values- LEARN BY HEART!!

	0°	30°	45°	60°	90°
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$undefined$

Online clips

U605, U283, U545, U627

Angles in Polygons

Component Knowledge

- Recognise and name different polygons
- Understand the difference between regular and irregular polygons
- Calculate and use the sum of interior angles
- Know that the sum of any exterior angles of any polygon is 360°
- Know that the interior + exterior angle is 180°

Key Vocabulary

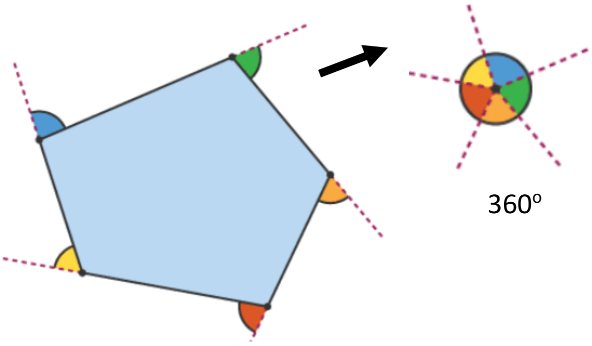
Interior angles	The angles inside the shape
Exterior angles	The angles between the side of a shape and a line extended from the adjacent side
Sum	Total – to add all the angles together
Polygon	A 2D closed shape made with straight lines
Regular	When all the sides are the same length and all angles are the same
Irregular	Shape with sides of different lengths and angles of different sizes

Exterior angles

The **sum of exterior angles** in any polygon is **360°**

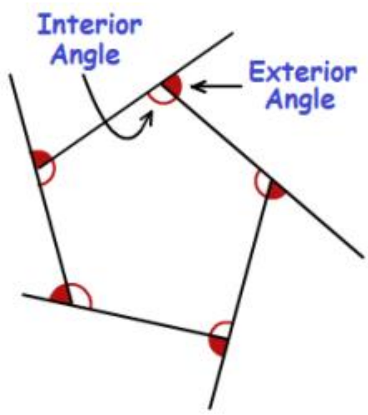
The size of each exterior angle in a regular polygon is **360° ÷ number of sides**

This can be rearranged to
number of sides = 360 ÷ angle

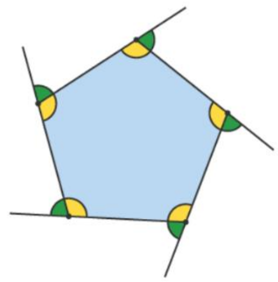


Interior and Exterior angles

Interior + exterior angle = 180°



Example 1 – Calculate the interior and exterior angle of a regular pentagon.



Exterior angle

= 360 ÷ 5 = 72°

Interior angle = 180 -72 = 108°

Example 2 – A regular polygon has exterior angles of 20°. How many sides does it have?

Exterior angle = 360 ÷ number of sides

Number of sides = 360 ÷ 20 = 18

= 18 sides

Interior angles in regular polygons

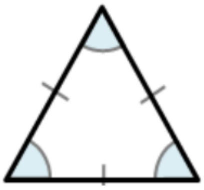
Sum of interior angles = (n – 2) x 180

Where n is the number of sides.

Each interior angle on a regular shape =

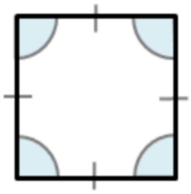
Total interior angles ÷ number of sides

Triangle



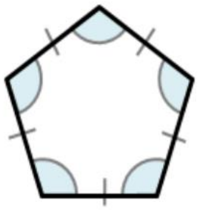
Number of sides	3
Sum of interior angles	180º
Size of each interior angle	60º

Square



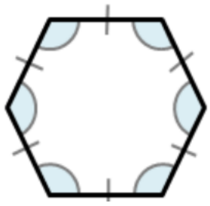
Number of sides	4
Sum of interior angles	360º
Size of each interior angle	90º

Pentagon



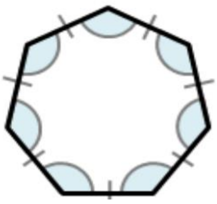
Number of sides	5
Sum of interior angles	540º
Size of each interior angle	108º

Hexagon



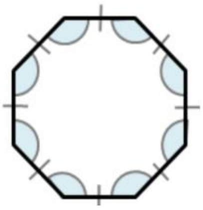
Number of sides	6
Sum of interior angles	720º
Size of each interior angle	120º

Heptagon



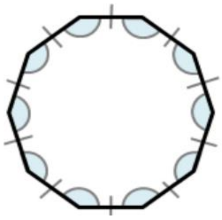
Number of sides	7
Sum of interior angles	900º
Size of each interior angle	128.6º (1dp)

Octagon



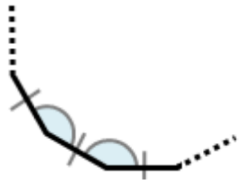
Number of sides	8
Sum of interior angles	1080º
Size of each interior angle	135º

Decagon



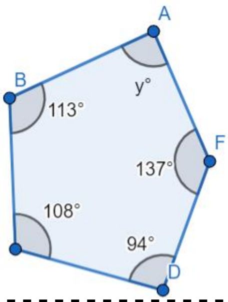
Number of sides	10
Sum of interior angles	1440º
Size of each interior angle	144º

***n* Sided Shape**



Number of sides	n
Number of interior angles	(n- 2)x 180º
Size of each interior angle	$\frac{(n-2) \times 180°}{n}$

Irregular polygons



Example –Find the value of y

5 sides, irregular polygon

Sum of interior angles = (5 -2) x180 = 540º

113 + 108 + 137 + 94 = 452

540 – 452 = 88 **x = 88º**

Online clips

U628, U732, U329, U427

Algebraic Vocabulary

Component Knowledge

- Understand the difference between the various algebraic words
- Understand how each previous word builds on to the next

Key Vocabulary

Variable	A quantity that can take on many values denoted by a symbol or a letter
Term	Is a single variable or number or variables and numbers multiplied together.
Expression	A group of numbers, letters and operational symbols, e.g. 2x + 3y -8
Equation	A number statement with an equals sign (=). Expressions on either side of the equals sign are of equal value, e.g. a + 14 = 20 or 2(x + 12) = 44 or x + 5 = 2x + 3
Formula	A special type of equation that shows the relationship between different variables. They tend to describe real-world situations. Plural is formulae.
Identity	An equation where both sides are identical whatever the value of the variable

A **variable** is a symbol (often a letter) that is used to represent an unknown.

E.g. x or y or a etc.

Variables can also have exponents (can be raised to a certain power.

E.g. x^2

A coefficient is the value that is before a variable. It tells us how many lots of the variable there is.

E.g. $x + x + x + x + x = 5 \times x = 5x$

The coefficient here is 5.

An **algebraic term** is either a single number or a variable.

e.g. '3' or 'x' or 'h'

A term can also be a number and a variable multiplied together.

e.g. 2a or 6y or 4xy

When 2 or more algebraic terms are added (or subtracted) they form an expression.

Formula/Formulae

A formula is a special type of equation that shows the relationship between different substituted variables. Formulas are often used in geometry to find area and volume.

Area of triangle = (base × height) ÷ 2

Area of rectangle = length × width

(12.5 × hours worked) + 25 = cost of job

Algebraic identities use the '≡' symbol. It is like an equal's sign, but it means identical to. No matter what the value of the variable this will always be true.

e.g. 2x = x + x

An **algebraic expression** is a single term or a set of terms that are combined using addition (+), subtraction (-), multiplication (x) and division (÷)

Examples

$3x$ $\quad$ $2x + 3y$ $\quad$ $2 - 5y^2$ $\quad$ $2x + 3y - 5$

An expression that contains two terms is called a binomial.

Equations are mathematical expressions which contain one or more variables and an equals sign.

Examples $\quad$ $3x - 5 = 7$ $\quad$ $\frac{4(x-2)}{5} = 8$ $\quad$ $x^2 = 9$ $\quad$ $2x^2 - 3x - 5 = 0$

We can solve an equation to find the value of the variable(s).

Example $\quad$ Solve $4x + 3 = 23$

$$4x + 3 = 23$$
$$-3 \qquad -3$$
$$4x = 20$$
$$\div 4 \qquad \div 4$$
$$x = 5$$

<u>Online clips</u>

M813, M830

Collecting Like terms

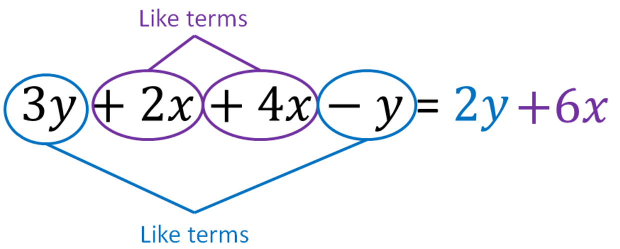
Component Knowledge

- Recognise terms in algebra
- Use of positive and negative directed numbers

Key Vocabulary

Variable	A **Variable** is a symbol for a number we don't know yet. It is usually a letter like x or y
Term	A **Term** is either a single number or a variable (x), or numbers and variables multiplied together ($5y$).
Expression	An **Expression** is a group of terms (the terms are separated by + or − signs) (eg, $5y + 6x - 8y$)
Simplify	reducing the expression/fraction/problem in a simpler form.

Collecting like terms : We collect like terms to simplify an expression. We look at terms which share the same variable



In this example:

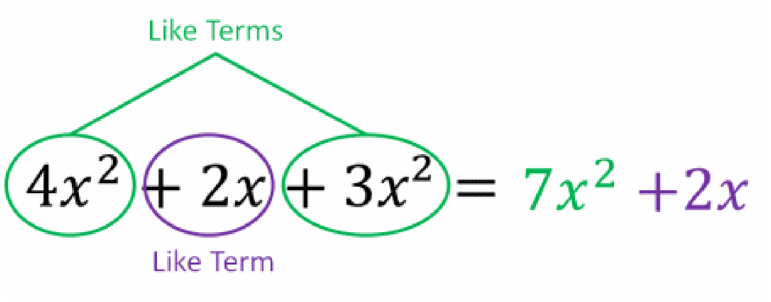
We collect all the x variables : $2x+4x = 6x$

AND

Collect all the y variables: variables : $3y - y = 2y$

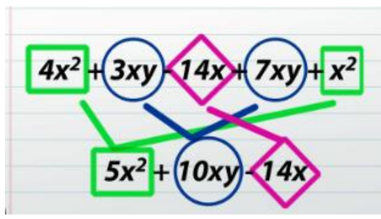
Collecting like terms – example 2

When collecting like terms, it is important to find the same terms and combine them to simplify the algebraic expression. We need to be able to recognise that x is different to x^2



Handy Hint:

It helps if you can visually see the different terms before you collect them. Using a different coloured pen, highlighter or shape works!



Online Clips

M795, M531, M949

Simplifying Expressions

Component Knowledge

- Law of indices
- Collecting like terms
- Recognise Algebraic terms and expressions

Key Vocabulary

Terms	In Algebra a term is either a single number or variable
Expression	Numbers, symbols and operators grouped together to show the value of something
Simplify	Reducing the expression/fraction to a simpler form.

Simplifying Terms - Multiplying:

Algebraic terms can be multiplied to give a simplified term. We focus on the number first, and then the variable (x or y), often using laws of indices.

Important – we always write terms in alphabetical order

Example	Answer
$2x \times 3 =$	$6x$
$4a \times 5b =$	$20ab$
$y^2 \times y^3 =$	$y \times y \times y \times y \times y = y^5$
$2ab \times 8cd =$	$2 \times 8 \times a \times b \times c \times d = 16abcd$
$a^5 b^3 \times a^4 bc^2 =$	$a^9 b^4 c^2$

Remember, any number to the power 0 is always 1

Simplifying Terms - Dividing:

Algebraic terms can be divided to give a simplified term. We focus on the number first, and then the variable (x or y), often using laws of indices.

Important – we should always write the division as a fraction,

e.g. $12a \div 6 = \frac{12a}{6}$

Example	Answer
$\frac{12a}{6} =$	$2a$
$\frac{18x}{24} =$	$\frac{3x}{4}$
$y^5 \div y^3 =$	$\frac{y \times y \times y \times y \times y}{y \times y \times y} = y^2$
$15a^4 \div 3a^2 =$	$\frac{15 \times a \times a \times a \times a}{3 \times a \times a} = 5a^2$
$a^3 \div a^3 =$	1

Online Clips

M795, M531, M120

Index laws

Component Knowledge

- Evaluate negative powers
- Simplify indices using multiplication and division.

Key Vocabulary

Power	A notation and word used to show repeated multiplication of the same number
Index	Another term used for power
Reciprocal	The *reciprocal* of a (non-zero) quantity is its inverse – informally, the fraction 'one over that number'

Negative indices

What pattern can you see?

2^4	16	
		÷ 2
2^3	8	
		÷ 2
2^2	4	
		÷ 2
2^1	2	
		÷ 2
2^0	1	
		÷ 2
2^{-1}	$\frac{1}{2}$	
		÷ 2
2^{-2}	$\frac{1}{4}$	

Step 1: Write as a fraction with 1 as the numerator.

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Step 2: Change the sign of the power.

The general rule for negative indices is: $a^{-x} = \frac{1}{a^x}$

(A negative power is equivalent to the *reciprocal* of a positive power)

$$3^{-3} = \frac{1}{3^3} = \frac{1}{27} \qquad \left(\frac{1}{5}\right)^{-1} = \frac{1}{\frac{1}{5}} = 5$$

Multiplying terms with indices

$$2^4 \times 2^3$$

Note that the *base* numbers are the same, 2

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 2^7$$

$$a^x \times a^y = a^{x+y}$$

This is the general rule: when multiplying same base numbers with powers, the powers are added to simplify

Misconception alert!

This rule only applies when the base numbers are equal. For example, there is no way of simplifying $2^3 \times 3^4$.

$$2^3 \times 3^4 \neq 6^7$$

$$3^3 \times 3^7 \times 3^{10}$$
$$= 3^{3+7+10}$$
$$= 3^{20}$$

$$2^5 \times 2^{-3}$$
$$= 2^{5+(-3)}$$
$$= 2^2$$

Dividing terms with indices

Note that the *base* numbers are the same, 2

$$\frac{2^6}{2^3} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2}}$$

$$= 2 \times 2 \times 2 \quad = 2^3$$

$$\boxed{\frac{a^x}{a^y} = a^{x-y}}$$

This is the general rule: when dividing same base numbers with powers, the powers are subtracted to simplify

As with the rule for multiplying, this rule only applies when the base numbers are equal. For example, there is no way of simplifying $10^5 \div 2^4$.

(You can *evaluate* it by calculating $100000 \div 16$ but it is not the same as 5^1)

$7^{12} \div 7^4$

$= 7^{12-4}$

$= 7^8$

$8^{15} \div 8^{-3}$

$= 8^{15-(-3)}$

$= 8^{18}$

Power of a power

$$(5^3)^2$$

$$5^3 \quad \times \quad 5^3$$

$$5 \times 5 \times 5 \quad \times \quad 5 \times 5 \times 5$$

$$= 5^6$$

$$\boxed{(a^x)^y \; = a^{xy}}$$

This is the general rule: a power of a power can be simplified by *multiplying* the powers

When simplifying (or evaluating) a power of a bracket, remember that the power applies to all factors inside the bracket.
In this example, both the coefficient 3 and the term b^4 must be squared.

$(3b^4)^2$

$= 3b^4 \times 3b^4$

$= 3 \times 3 \times b^4 \times b^4$

$= 9b^8$

$(3b^4)^2$

$= 3^2 \times b^8$

$= 9b^8$

Example using all rules of indices

$$\frac{5^3 \times 5^4}{(5^2)^5} = \frac{5^3 \times 5^4}{5^{10}} = \frac{5^7}{5^{10}} = 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

Power of a power — Multiplication rule — Division rule — Negative power

Online clips

U235, U694, U662

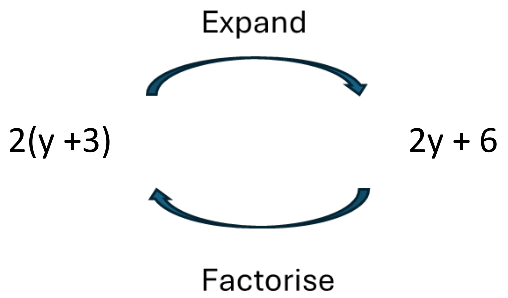
Expanding single brackets

Component Knowledge

- Multiply terms together to remove a bracket from an expression
- Collect like terms together when expanding multiple single brackets

Key Vocabulary

Expression	Numbers, symbols and operators grouped together that show the value of something
Expand	Remove the brackets by multiplying terms together
Simplify	Collect like terms together
Term	Either a single number or variable or numbers and variables multiplied together



Expanding is the opposite of factorising. When we expand, we are multiplying terms together to remove the brackets.

Examples

Expand and simplify where appropriate

1) $7(3 + a) = 21 + 7a$

2) $2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a$

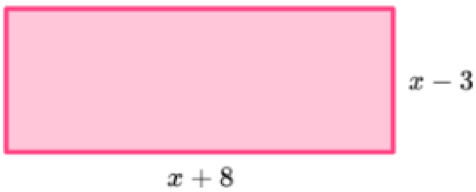
$= 5a + 16$

Note – collect like terms to simplify

To expand, we multiply all the terms inside the bracket by the term in front of the bracket

One use of brackets in maths in maths is to group items together, another is to give information about the order of operations.

Here is a rectangle.



Its perimeter is:

$(x+8)+(x-3)+(x+8)+(x-3)$

Here the brackets are used to group the terms so that the expressions for the sides are clear.

The perimeter can also be written as: $2(x+8)+2(x-3)$

Here brackets are needed to preserve that the whole expressions for the sides are doubled to find the perimeter of a rectangle.

Online clips

U179, U105

Expanding Double Brackets

Component Knowledge

- To use algebraic notation when multiplying terms.
- To be able to expand double brackets and simplify where necessary.
- Use identity notation correctly.

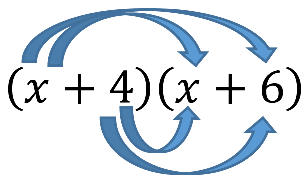
Key Vocabulary

Expand	Multiplying out a bracket.
Term	Either a single number or variable, or the product of several numbers or variables.
Collecting like terms	Simplifying an expression by grouping the same type of terms together.
Identity	An equality that relates one variable to another. It will be equal for ALL values of the variable, unlike an equation which gives a single solution.

Expanding double brackets

Expanding double brackets is long multiplication using algebraic terms as well as numerical values. There are 2 common ways of completing this.

Example 1 -Expand $(x+4)(x+6)$



$$(x+4)(x+6)$$

We multiply all terms together (this can be known as FOIL method):

$$x \times x = +x^2$$

$$x \times 6 = +6x$$

$$4 \times x = +4x$$

$$4 \times 6 = +24$$

$$(x+4)(x+6) \equiv +x^2 + 6x + 4x + 24$$

We now collect like terms:

$$\equiv +x^2 + 10x + 24$$

Example 1 -Expand $(x+4)(x+6)$

We can also use an area model (also known as the grid method).

X	x	+4
x	$+x^2$	+4x
+6	+6x	+24

We have still multiplied all the terms together, like the previous method, but they remain in the grid. We can see all 4 terms in the expanded expression in the grid:

$(+x^2 + 6x + 4x + 24)$.

We now collect like terms:

$$(x+4)(x+6) \equiv +x^2 + 10x + 24$$

Example 2 -Expand $(x+4)(x-6)$

$$(x+4)(x-6)$$

We multiply all terms together

$x \times x = +x^2$

$x \times -6 = -6x$

$4 \times x = +4x$

$4 \times -6 = -24$

Note: be careful when multiplying and adding with negatives!

$(x+4)(x+6) \equiv +x^2 - 6x + 4x - 24$

We now collect like terms:

$\equiv +x^2 - 2x - 24$

Example 2 -Expand $(x+4)(x-6)$

X	x	+4
x	$+x^2$	+4x
-6	-6x	-24

We can see all 4 terms in the expanded expression in the grid:

$+x^2 - 6x + 4x - 24$

We now collect like terms:

$(x+4)(x-6) \equiv +x^2 - 2x - 24$

Example 3 -Expand $(x-4)(x-6)$

$$(x-4)(x-6)$$

$x \times x = +x^2$

$x \times -6 = -6x$

$-4 \times x = -4x$

$-4 \times -6 = +24$

$(x-4)(x-6) \equiv +x^2 - 6x - 4x + 24$

We now collect like terms:

$\equiv +x^2 - 10x + 24$

Example 3 -Expand $(x-4)(x-6)$

X	x	-4
x	$+x^2$	-4x
-6	-6x	+24

We can see all 4 terms in the expanded expression in the grid:

$+x^2 - 6x - 4x + 24$

We now collect like terms:

$(x-4)(x-6) \equiv +x^2 - 10x + 24$

Online clip

M960

Factorise Linear expressions

Component Knowledge

- Factorise an expression with a numerical common factor.
- Factorise an expression with a variable (letter) as the common factor

Key Vocabulary

Factor	A number or quantity that when multiplied with another produces a given number or expression.
Factorise	The reverse of expanding. Factorising is writing an expression as a product of terms by 'taking out' a common factor.
Expression	A mathematical statement written using symbols, numbers or letters.

Factorising Examples

- Factorising is the opposite of expanding a bracket.
- Find the largest common factors of all terms and divide by these.
- The factors are put in front of the bracket.

Example 1

Factorise fully:

12y + 4

What is common to both? **4**

4 goes on the outside of the bracket

4 (3y + 1)

Check your answer by expanding the bracket.

Example 2

Factorise fully:

$$18a^2 - 4a$$

What is common to both? **2 and** a

2 a goes on the outside of the bracket

2 a **(9** a **- 2)**

Check your answer by expanding the bracket.

Online clips

U365

Solving linear equations

Component Knowledge

- To be able to solve one-step equations.
- To be able to solve two-step equations.
- To be able to solve three-step equations
- To be able to form and solve equations

Key Vocabulary

Operation	Common operations are addition, subtraction, multiplication and division.
Inverse	The opposite operation of another function.
Equation	a mathematical statement that shows that two mathematical expressions are equal
Solve	To find the solution

One- step equations

To solve a one-step equation, you need to do the inverse operation.

÷5	5x	=	30	÷5
	x	=	6

÷5: 5x = 30 → x = 6 (÷5)

The inverse of multiplying is dividing.

We divide 30 by 5.

+3: x – 3 = 7 → x = 10 (+3)

The inverse of subtracting is addition.

We add 3 to 7.

-5: x + 5 = 9 → x = 4 (-5)

The inverse of addition is subtraction.

We subtract 4 from 9.

x 3: $\frac{x}{2}$ = 3 → x = 6 (x 3)

The inverse of dividing is multiplying.

We multiply 2 by 3.

Two- step equations

To solve a two-step equation, we need to complete 2 inverse calculations in a specific order.

6x + 3 = 32

-3 — The inverse of adding 3 is subtracting 3

6x = 30

÷ 6 — The inverse of multiplying by 6 is dividing by 6

x = 5

$\frac{x-5}{3}$ = 4

x3 — The inverse of dividing by 3 is multiplying by 3

x – 5 = 12

+ 5 — The inverse of subtracting 5 is adding 5

x = 14

Three - step equations

To solve a three – step equation, we need to complete 3 inverse calculations in a specific order.

$\frac{2x}{7} + 2 = 12$

-2 — The inverse of adding 2 is subtracting 2

$\frac{2x}{7} = 10$

x 7 — The inverse of dividing by 7 is multiplying by 7

2x = 70

÷ 2 — The inverse of multiplying by 2 is dividing by 2

X = 35

$\frac{8x}{3} - 9 = 7$

+ 9 — The inverse of subtracting 9 is adding 9

$\frac{8x}{3} = 16$

x 3 — The inverse of dividing by 3 is multiplying by 3

8x = 48

÷ 8 — The inverse of multiplying by 8 is dividing by 8.

x = 6

Forming and solving equations

I think of a number, multiply it by 3 and add 5. The answer is 29. What number did I think of?

Write the unknown number as a letter like x

Multiply x by 3 to get 3x

Add 5 to get 3x + 5

Put equal to 29 to get an equation to solve

3x + 5 = 29

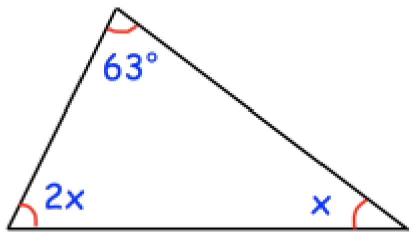
Solve

3x + 5 = 29 (-5)

3x = 24 (÷ 3)

x = 8

Form and solve an equation to find the size of the angle labelled x.



1st step: form an equation x + 2x + 63° = 180°

3x + 63° = 180°

2nd step: solve the equation 3x + 63° = 180° (-63)

3x = 117° (÷ 3)

x = 39°

3rd step: show your final answer <u>Angle x = 39°</u>

Online clips

U755, U325, U599

Changing the subject

Component Knowledge

- Use inverse operations to change the subject of a formula
- Rearranging simple and harder formula
- Use the order of operations to rearrange

Key Vocabulary

Rearrange	Change the order of
Inverse	The opposite (adding is the inverse of subtracting)
Operation	A mathematical process that produces an output (+, -, x, ÷)
Term	A part of an equation (2, a and 2a are all terms)
Formula	A fact or rule that relates two or more quantities
Subject	The beginning of a formula/equation

Rearrange to make r the subject of the formula:

X3 $Q = \frac{2r-7}{3}$ X3

+7 $3Q = 2r - 7$ +7

÷2 $3Q + 7 = 2r$ ÷2

$\frac{3Q+7}{2} = r$

Rearranging Formulae

Make u the subject: $v = u + at$

Change the order of the terms so 'u' is on its own

-at -at

$v - at = u$

so $u = v - at$

Make m the subject: $I = mv - mu$

If the letter appears twice you will need to factorise

÷ (v – u) $I = m(v - u)$ ÷ (v – u)

$I \div (v - u) = m$

$m = \frac{I}{v-u}$

e.g. make c the subject of the formula

$m = 5(c - 1)$

There are 2 options here:

Method 1: expand the bracket first

expand $m = 5(c - 1)$ expand

+5 $m = 5c - 5$ +5

÷ 5 $m + 5 = 5c$ ÷ 5

$\frac{m+5}{5} = c$

Method 2: divide by the coefficient first

÷ 5 $m = 5(c - 1)$ ÷ 5

+ 1 $\frac{m}{5} = c - 1$ + 1

$\frac{m}{5} + 1 = c$

Tip – examiners tell schools that method 1 usually has a higher success rate in an exam than method 2 does!

When the subject appears more than once in a formula, collect like terms together and factorise using the subject as the factor

Online clips

U675, U181

Pie charts

Component Knowledge

- Calculate angles in a pie chart
- Draw a pie chart from a table
- Interpret pie charts using fractions
- Interpret pie charts using angles

Key Vocabulary

Angle	The amount of turn between 2 lines.
Pie chart	A chart that displays data proportionally.
Protractor	Equipment used to measure and draw angles

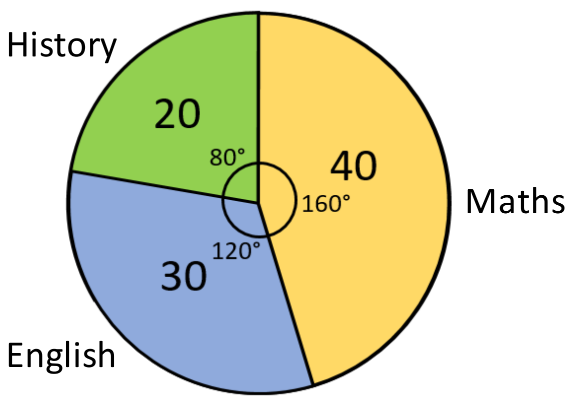
Drawing pie charts

How many degrees for one person? $\frac{360}{90} = 4^\circ$

360 ÷ total = degrees for one person. In this example one person is 4°.

Subject	Number of Students	Calculation	Angle
Maths	40	$40 \times 4^\circ$	160°
English	30	$30 \times 4^\circ$	120°
History	20	$20 \times 4^\circ$	80°
Total	90		360°

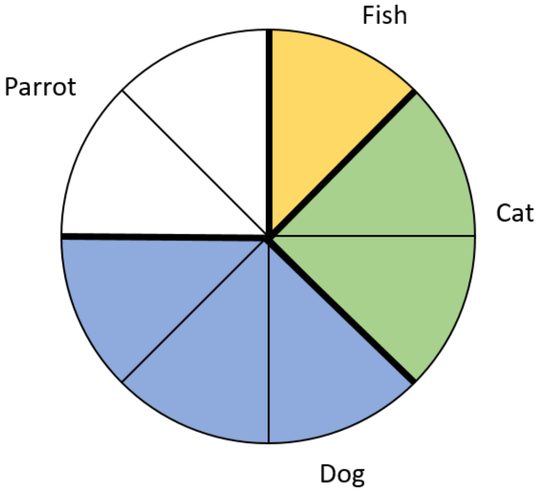
Multiply number of students by 4° to get the angle.



Draw the angles onto the pie chart. Label each part with what it is (subject in this example) and how many it represents (40 for Maths in this example).

Interpret pie charts (fractions)

A class of **32 students** were surveyed to find their **favourite pet**.
The **pie chart** shows the total answers. How popular was each animal?



The pie chart is split into 8 pieces, so each sector is worth $\frac{1}{8}$ of 32 = 4

Fish: $\frac{1}{8}$ of 32 = 4

Cat: $\frac{2}{8}$ of 32 = 8

Dog: $\frac{3}{8}$ of 32 = 12

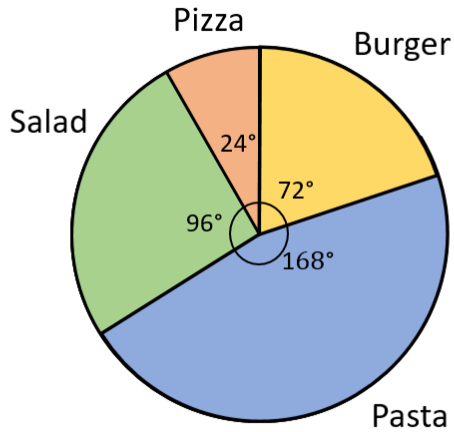
Parrot: $\frac{2}{8}$ of 32 = 8

Check that the totals add up to the original total in the question.
(4 + 8 + 12 + 8 = 32)

Interpret pie charts (angles)

150 students were surveyed about their favourite food.

Favourite Food	Angle	Calculation	Frequency
Burger	72°	$\frac{72}{360} \times 150$	30
Pasta	168°	$\frac{168}{360} \times 150$	70
Salad	96°	$\frac{96}{360} \times 150$	40
Pizza	24°	$\frac{24}{360} \times 150$	10



To calculate the frequency from a pie chart when you are given the angle, you do the opposite of what you do to calculate the angle.

Angle ÷ 360 × total frequency

Online clips

M574, M165